Module 7

Mathematics 6

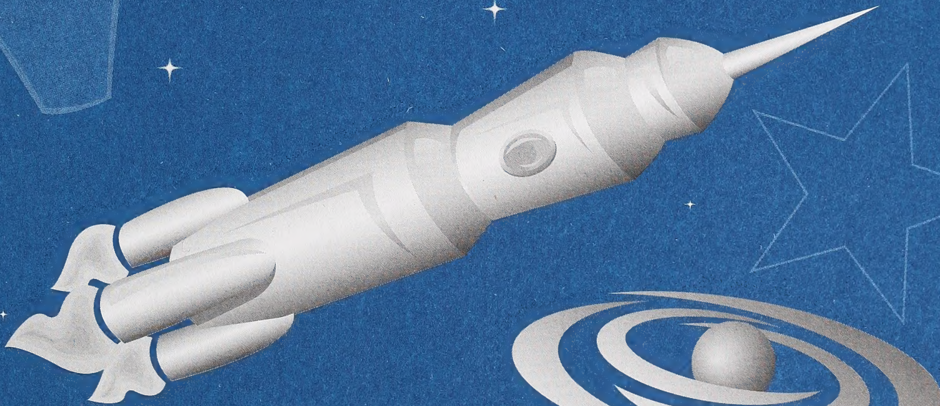
Transformations

Learning Technologies Branch

Alberta LEARNING

Mathematics 6

Module 7
Transformations



Learning Technologies Branch

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Mathematics 6
Module 7: Transformations
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

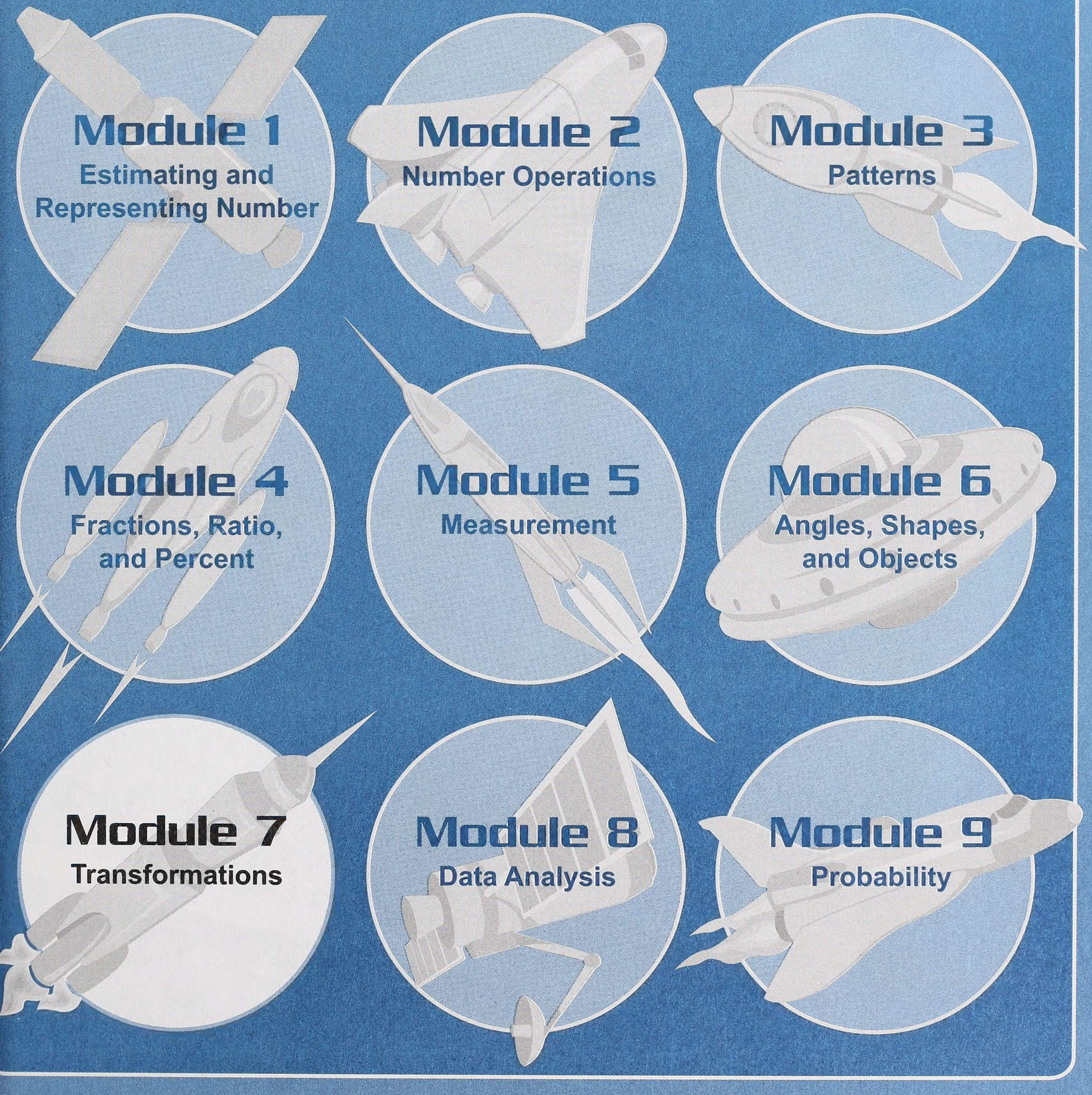
You may find the following Internet sites useful:

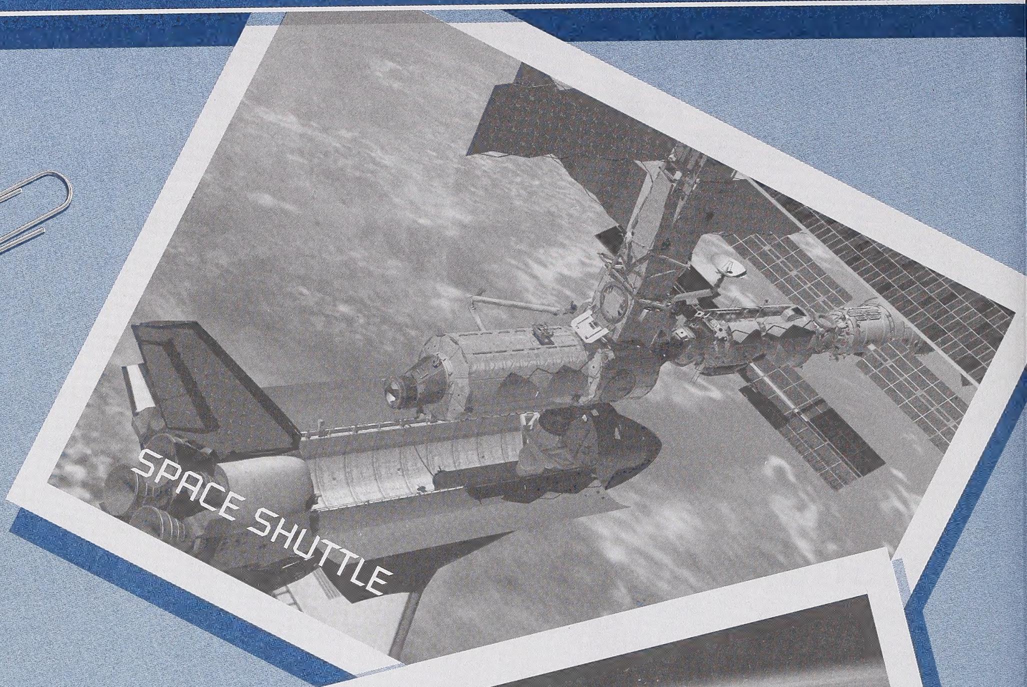
- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

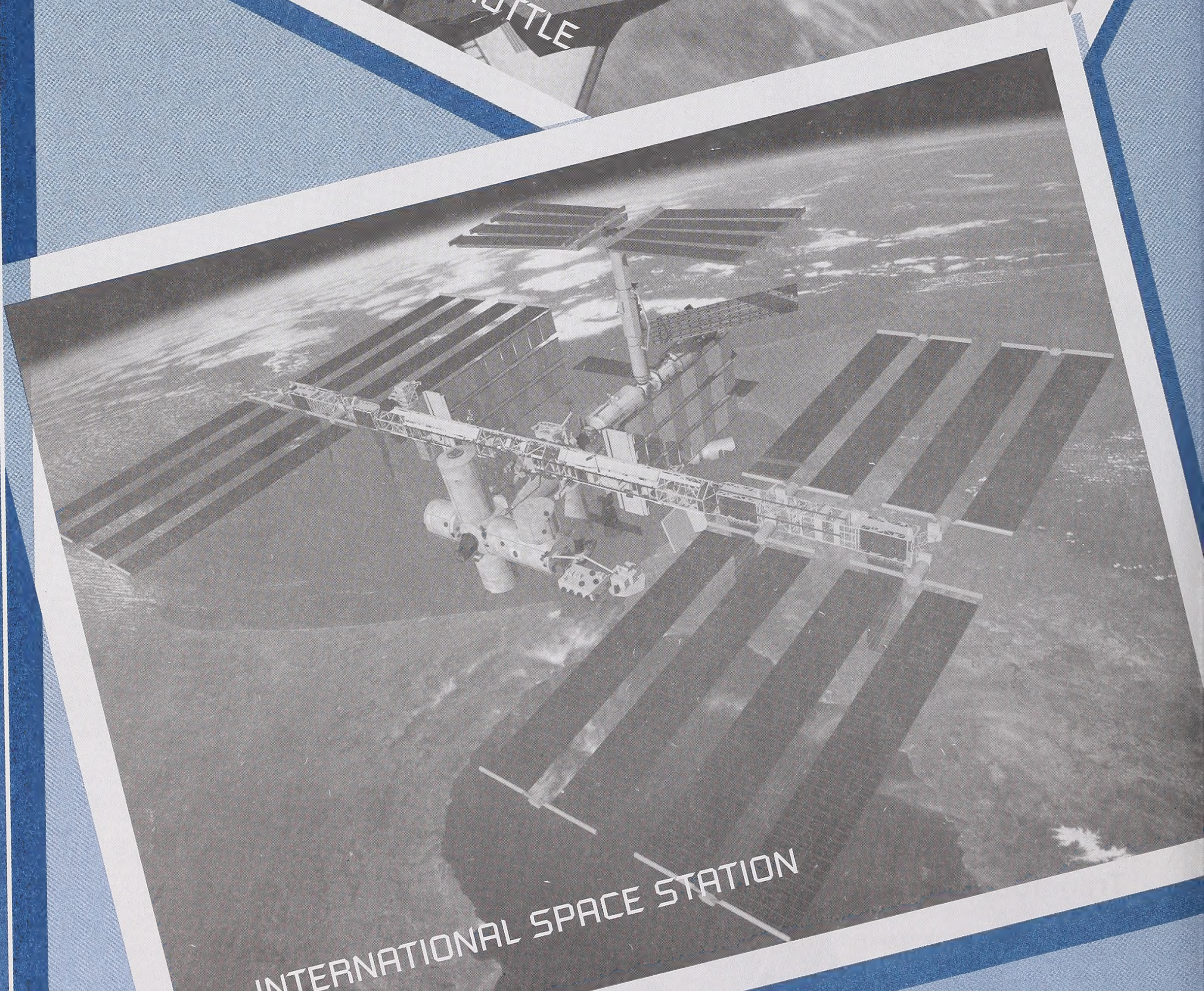
The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome to Mathematics 6

Mathematics 6 contains nine modules.
You should work through the modules in order (from 1 to 9) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.




SPACE SHUTTLE


INTERNATIONAL SPACE STATION

Adventures in Outer Space

Matthew: Wow, what a wonderful experience it was meeting Colonel Chris Hadfield at the Odyssium! He gave a presentation here in Edmonton on July 9, 2001, and talked about his adventures in space, including his mission aboard the Space Shuttle *Endeavor* to attach Canadarm2 to the International Space Station.

It's too bad you missed it, Kylee. You were away visiting your grandmother in Slave Lake.

Kylee: My trip was great, but I sure wish I could have heard Colonel Hadfield talk about being the first Canadian to walk in space. But I've got great news for you, Matthew! Commander Claire from the International Space Station is coming to town, and you and I will be spending some time with her.

I can't wait to hear about her adventures in space!

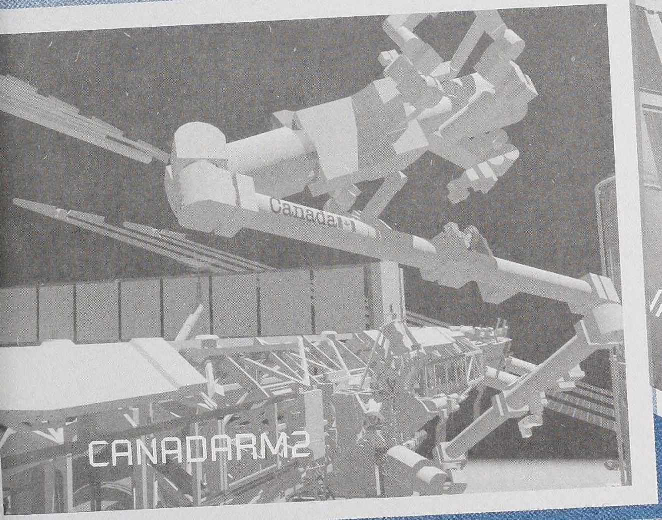


Table of Contents


Commander Claire

Course Features

Take the time to look through the Student Module Booklets and the Assignment Booklets and notice the following design features:

- Each module has a Module Overview, Module Summary, and Review.

- Each module has several lessons. Each lesson focuses on a big idea that is central to the topic being learned in the module.

- Each lesson has several activities. An activity in each lesson is related to the Adventures in Outer Space theme.

- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.

- Each module has special exercises that focus on certain mathematical skills. The Numbers in the News project involves a scavenger hunt for samples of math in everyday life. The Keystrokes exercise introduces some “funky features” of the calculator that can be used to explore and practise important number ideas. Just the Facts gives you the opportunity to practise your basic number facts by doing a timed drill with your home instructor. The Mental Math exercise introduces an estimation skill or mental-computation strategy that you can use to sharpen your mental math skills.

- Each module references the Mathematics 6 Companion CD that includes additional material for review and mastery.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

In order to complete the course, you will need a copy of the Mathematics 6 textbook, *Quest 2000: Exploring Mathematics*, *Grade 6*, the soft-cover book *Quest 2000: Exploring Mathematics: Practice and Homework Book*, *Grade 6*, a basic four-operation calculator (such as the TI-108 calculator), and various manipulatives (base ten blocks and pattern blocks).

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

You will also need access to a computer to view material on the Mathematics 6 Companion CD.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.

	Use your calculator.		Use the Internet.
	Refer to the textbook or the Practice and Homework Book.		Use the Mathematics 6 Companion CD.

Assessment and Feedback

The Mathematics 6 course is carefully designed to give you many opportunities to discover how well you are doing. In every activity you will be asked to turn to the Appendix to check your answers. Completing the activities and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.

If you are having difficulty with an activity, refer to the Answer Key in the Appendix for hints or help. As well as giving suggested answers to the questions, the Answer Key gives you more information about the questions.

Twice in each module you will be asked to give your teacher your completed assignments to mark. Your teacher will give you feedback on how you are doing.

After your teacher marks an assignment, be sure to review your teacher's comments and correct any errors you made.

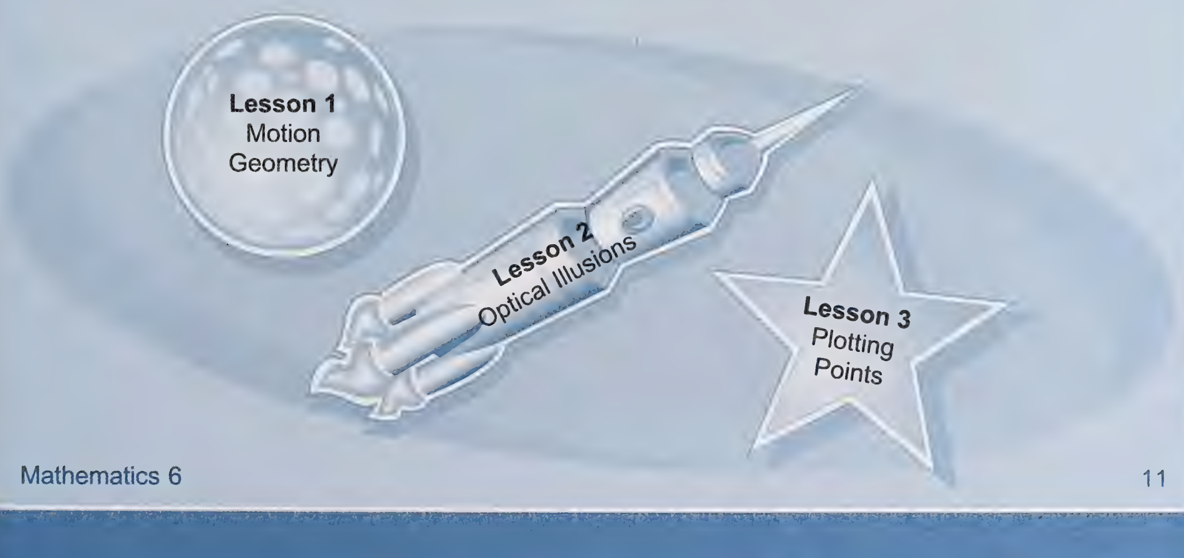
There will be a final test at the end of the course. You can prepare for the final test by completing the Review at the end of each module.

Module Overview

In the late nineteenth century, astronomers who were observing the surface of Mars through powerful telescopes thought they saw a network of canals. They thought these canals had been constructed by an ancient race of Martians in a desperate attempt to transport water throughout a dying planet.

In fact, these "canals" are constructed by the human mind as it tries to connect points and features on the surface of the planet. The canals are simply an optical illusion.

In this module you will encounter a variety of optical illusions—some that occur in nature and others that are created by people. Many of these illusions involve geometric shapes. You also will extend your knowledge of geometric shapes by exploring transformations, motion geometry, and the coordinate plane.



Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 7A

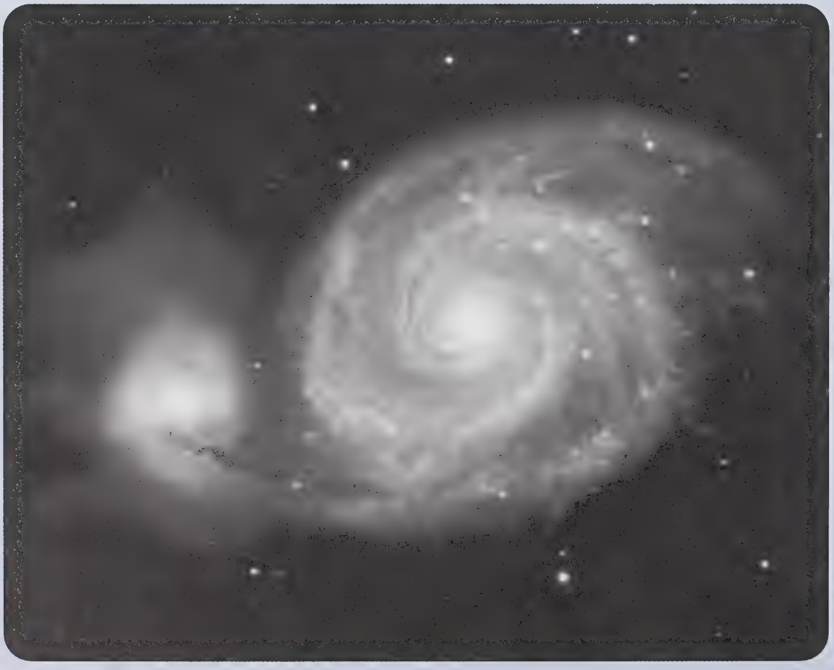
Lesson 1 Assignment	30 marks
Lesson 2 Assignment	30 marks

Assignment Booklet 7B

Lesson 3 Assignment	30 marks
Numbers in the News	10 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

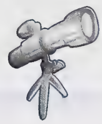
You will submit Assignment Booklet 7A to your teacher before you begin Lesson 3. You will submit Assignment Booklet 7B to your teacher at the end of this module.



Numbers in the News ...3...2...1...

Read through the following list before you begin Module 7. Begin by collecting samples of the ideas you already understand; others you may collect as you learn about them in the module. The samples you collect will depend on the newspapers or magazines you use.

Scavenger Hunt



Cut out articles or advertisements from newspapers or magazines that show transformations being used in different situations. Here are some suggestions of things to look for:

- pictures of tessellations
- pictures or drawings that show
  - slides
  - flips
- pictures of optical illusions that are
  - natural
  - created by humans
- diagrams or pictures that show a coordinate system

You will find further instructions for completing and submitting your project in Assignment Booklet 7B.

Lesson 1

Motion Geometry

Decorating eggs with intricate geometric designs has been a springtime tradition among the people of Russia and Ukraine for over 2000 years. Immigrants brought this art form to Canada and many of their descendants continue this custom. If you look carefully at the designs, you will notice a variety of repeated patterns that combine to create a visually pleasing effect. These repeated geometric forms that cover the surface are examples of tessellations.

In this lesson you will explore how slides, flips, and turns can be used to create tessellations with a variety of geometric figures. You will investigate why some figures tessellate and others do not. You will learn how to create interesting designs.

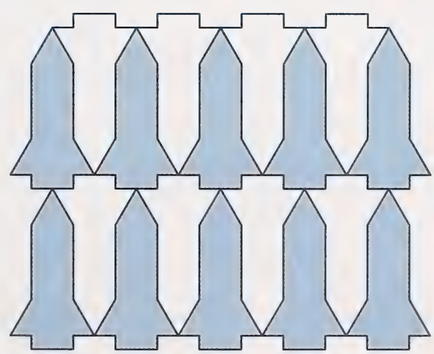
Activity 1

Today you will explore tessellations.

NASA

The Space Shuttle is truly an engineering marvel. When I return from a mission, I can't wait for the next one!

Commander Claire created a design to be used on fabric and paper. The picture at the right shows Claire's shuttle design in white and blue. Notice that slides and flips are used to make a tessellation with the shuttle design.



When a shape **tessellates**, it covers a surface without leaving any gaps or overlaps.

When a shape **slides**, it moves in a straight line without twisting or turning.

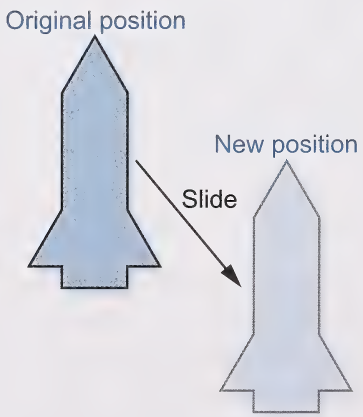
When a shape **flips**, it forms a mirror image.

Study the following example to review slides and flips.

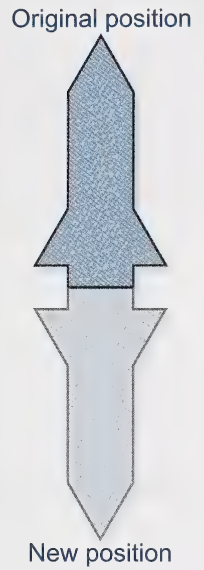
Example

Illustrate a slide and a flip using the following figure.

First, look at a slide. Move the figure in any direction without twisting or turning it.

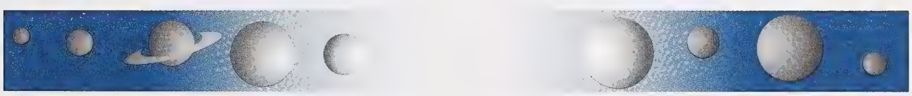


Next, look at a flip.



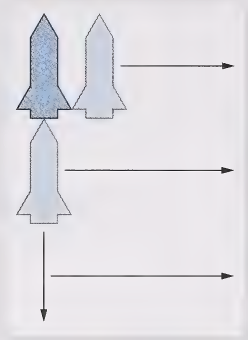
The two figures look like mirror images of each other.

You will use slides and flips in the questions in this activity.



Trace and cut out the shuttle design at the right, glue it onto light cardboard, and cut it out. Use your cardboard design to answer questions 1 to 3.

1. Place your cardboard design in the upper left corner of a sheet of paper, as shown. Slide the design across and down the sheet, and trace it each time. Colour each figure you trace.



2. a. What is the shape of the gaps formed between the figures you traced and coloured?

   b. What must you do with your cardboard figure to make it fit into one of the gaps?

3. a. On your tessellation, use a red pencil crayon to draw the **lines of symmetry**.

   b. Describe the pattern made by the lines of symmetry.

Check your answers on pages 78 and 79 in the Appendix.

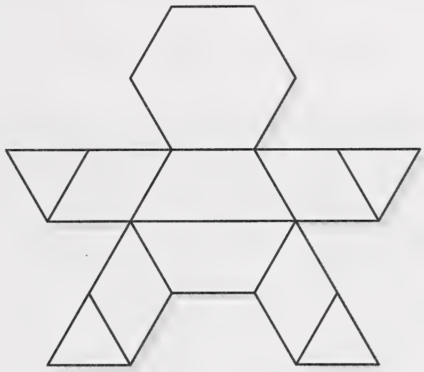
4. Claire wants to build a model of her shuttle design using pattern blocks. Show how she can use one triangle, two squares, and one trapezoid to make a tessellating design that very closely resembles the shuttle design.

5. Make a pattern block tessellation with your shuttle design from question 4. Can you do this using only slides? Explain.

6. Draw a picture of your tessellation from question 5 and colour it.

Check your answers on pages 79 and 80 in the Appendix.

7. Claire used pattern blocks to make an astronaut.



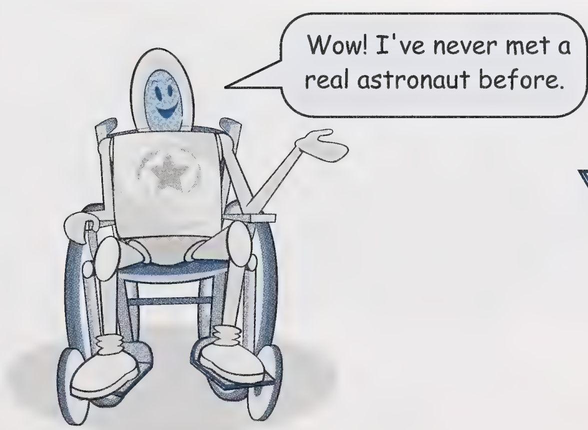
a. Colour the design to show the pattern blocks she used.

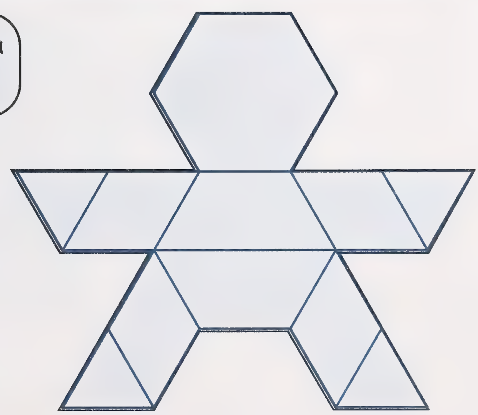
b. Each time she used a pattern block more than once, explain whether she needed to slide it, flip it, or turn it.

8. Sketch a picture to show how you can make Claire's astronaut design using only red trapezoids. Explain how you need to use slides, flips, and turns to make the design.

9. Use a sheet of isometric dot paper from the Appendix to see if Claire's astronaut design can be used to make a tessellation. Explain your results.

Check your answers on pages 80 and 81 in the Appendix.

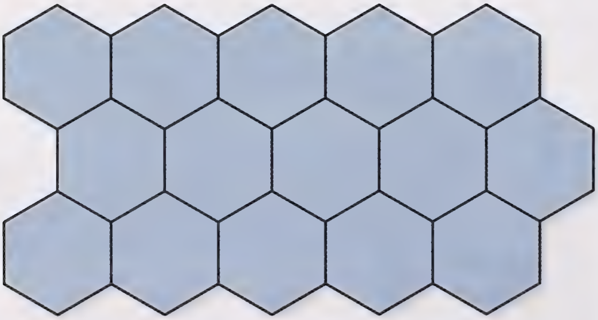




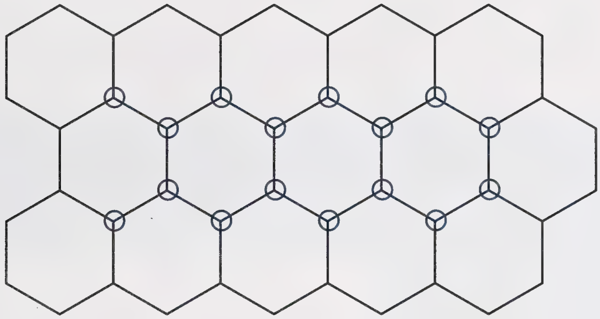
Activity 2

Today you will explore why some figures tessellate and some do not.

One way to tessellate shapes is to use slides and/or flips to put them in rows by joining their sides. By making the rows longer and adding more rows, you can cover a surface. This is shown with hexagons. Bees use tessellated hexagons to construct their honeycombs.



1. The following diagram shows all the vertices where the hexagons are joined in a tessellation.

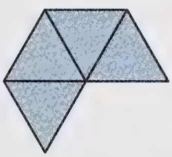


   a. What is the measure of each angle in a regular hexagon?

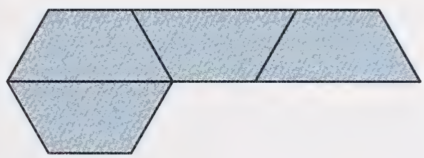
   b. What is the total measure of the three hexagon angles joined at each vertex?

2. The following pictures show pattern blocks joined at their sides. Copy and complete the pictures in your notebook to show three rows with five blocks in each row.

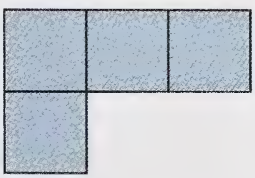
   a. green triangle



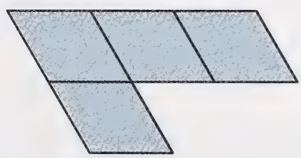
   b. red trapezoid



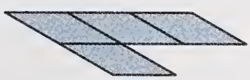
   c. orange square



   d. blue rhombus



   e. tan rhombus



3. Use the pictures you drew in question 2. Draw circles around each vertex where the shapes are joined.

4. Use your results from question 3 to copy and complete the following table in your notebook. The first one is done for you.

Pattern Blocks	Sum of the Angles at Each Vertex	Is the Polygon Regular?
Yellow Hexagons	$120° + 120° + 120° = 360°$	yes
Green Triangles		
Red Trapezoids		
Orange Squares		
Blue Rhombuses		
Tan Rhombuses		

5. Does a polygon have to be regular in order to tessellate? Explain.

Check your answers on pages 81 to 83 in the Appendix.

6. Draw an isosceles triangle on light cardboard and cut it out.

   a. Predict whether or not your isosceles triangle will tessellate.

   b. Verify your prediction by tracing your isosceles triangle in rows. Explain your answer.

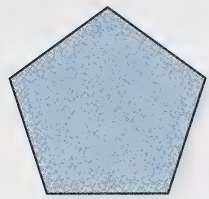
7. Draw a scalene triangle on light cardboard and cut it out.

   a. Predict whether or not your scalene triangle will tessellate.

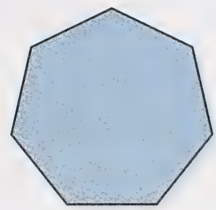
   b. Verify your prediction by tracing your scalene triangle in rows. Explain your answer.

8. Make stencils by tracing and gluing the following regular polygons on light cardboard and cutting them out.

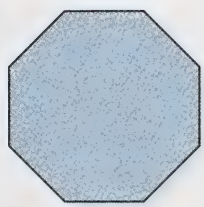
a. pentagon



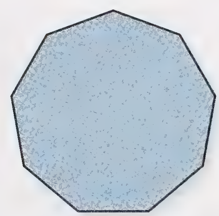
b. heptagon



c. octagon



d. nonagon



See if you can tessellate each regular polygon by tracing its template in rows.

9. a. Use your results from question 8 to copy and complete the following table in your notebook. One row has been done for you.

Regular Polygon	Size of Each Angle	Does It Tessellate?	Is the Size of Each Angle a Factor of 360°?
Triangle			
Square			
Pentagon			
Hexagon	120°	yes	yes ($3 \times 120° = 360°$)
Heptagon			
Octagon			
Nonagon			

b. The only regular polygons that tessellate are the equilateral triangle, the square, and the regular hexagon. Explain how this is related to the size of the angles in regular polygons.

Check your answers on pages 83 to 85 in the Appendix.

Activity 3

Today you will create tessellations using the techniques of Maurits Escher.

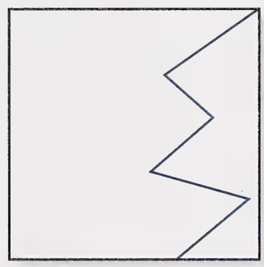
The Dutch artist Maurits Escher was not only a great artist, he was also a great mathematician! He created many interesting shapes that form tessellations.

If you have access to the Internet, you can find out more about Maurits Escher at the following website:

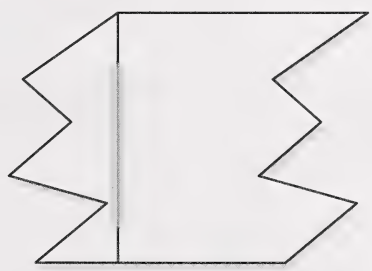
http://www.mcescher.com

One way to make an interesting tessellating shape is to begin with a simple tessellating shape, such as a square.

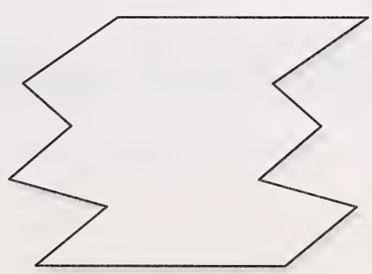
A square with a design drawn along its right side is shown. Trace the square and the design.



Cut out the design, slide it to the left side, and tape it as shown.



Glue your taped shape on light cardboard and cut it out. Your cardboard shape should look like this.



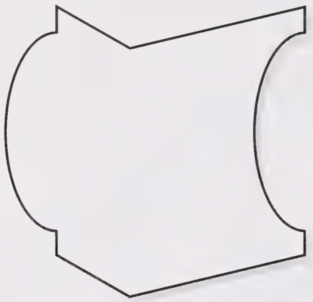
Use your cardboard shape to answer questions 1 and 2.

1. Predict the motion you can use to make a tessellation with your shape. Explain your answer.

2. Verify your prediction by tracing your cardboard shape to make a tessellation.

Check your answers on pages 85 and 86 in the Appendix.

Lei began with a square and made the tessellating shape shown below. Trace and glue her shape on light cardboard, cut it out, and use it to answer questions 3 to 5.

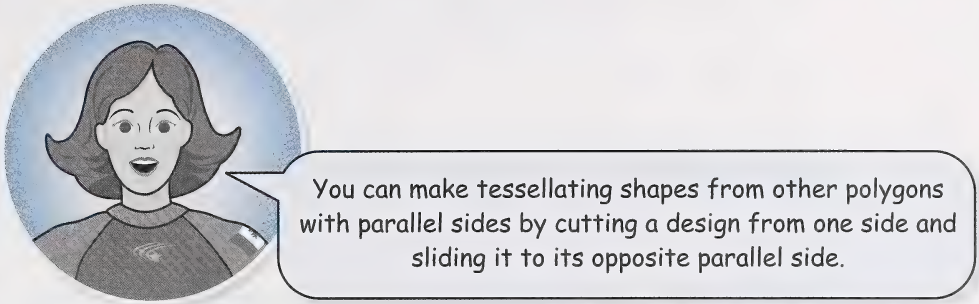


3. Explain how Lei made the shape. Draw a picture showing the starting square and use dotted lines on it to show the pieces she cut out and slid to their opposite sides.

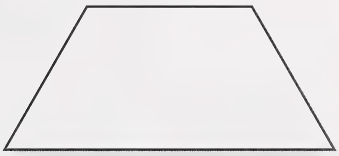
4. Predict the motion you can use to make a tessellation with Lei's shape. Explain your answer.

5. Verify your prediction by tracing your cardboard shape to make a tessellation.

Check your answers on page 86 in the Appendix.



6. A trapezoid is shown at the right.



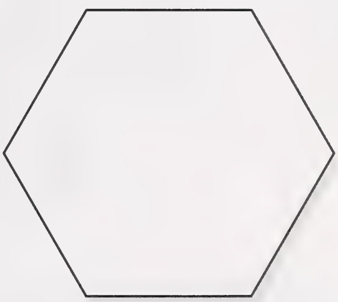
   a. Which of its sides are parallel?

   b. Trace and cut out the trapezoid. Make a tessellating shape with the trapezoid.

   c. Use your shape to make a tessellation.

   d. What motions did you use to make your tessellation?

7. A regular hexagon is shown at the right.



   a. How many pairs of parallel sides does it have?

   b. Trace and cut out the hexagon. Draw a different design on one side of each pair of parallel sides to make a tessellating shape.

   c. Use your shape to make a tessellation.

8. a. Explain why you cannot use the parallel-side method you used in question 7 to make a tessellating shape with a regular pentagon.

   b. Draw a pentagon that you can make into a tessellating shape.

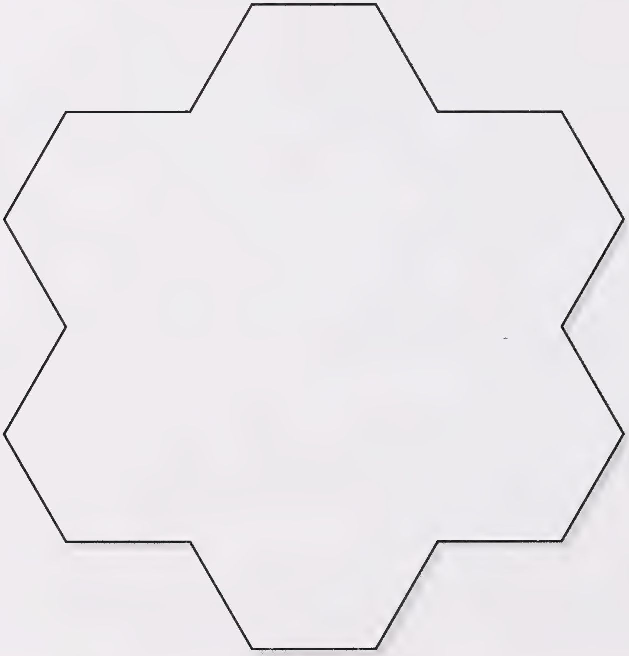
c. Use your shape to make a tessellation.

d. What motions did you use to make your tessellation?

Check your answers on pages 87 and 88 in the Appendix.

Challenge Activity

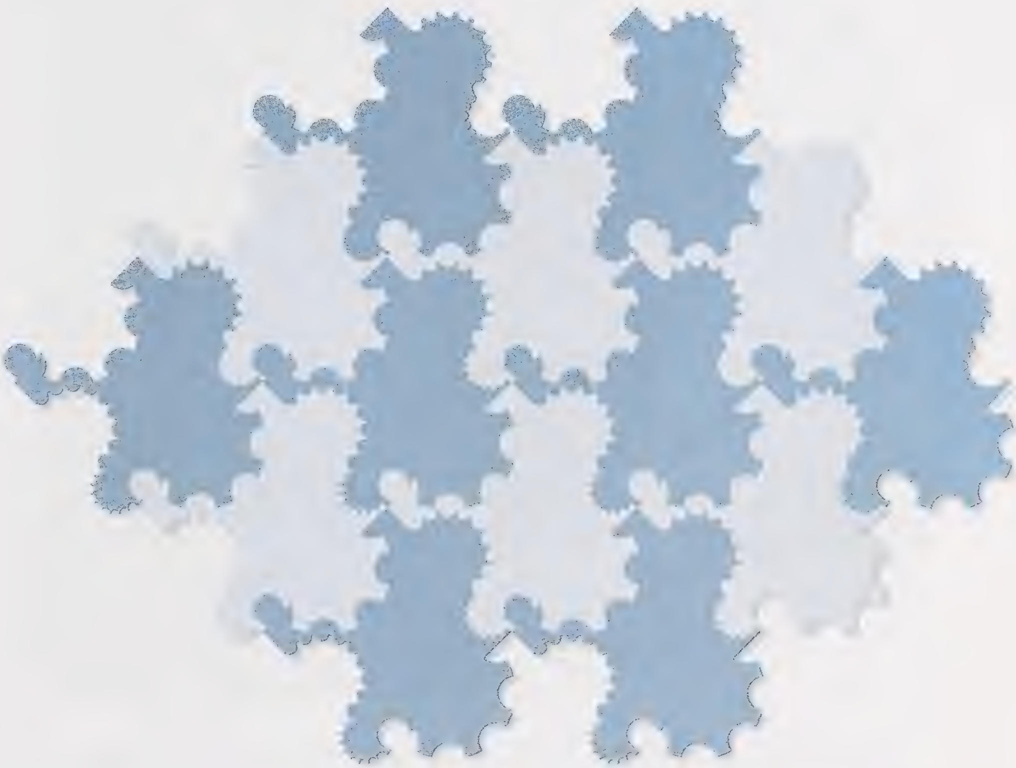
Remove the page of coloured hexagons from the Appendix. Cut out the hexagons and arrange them in the outline below so that each side of each hexagon matches up with the same colour in the hexagon next to it.



Check your answers on page 89 in the Appendix.

Conclusion

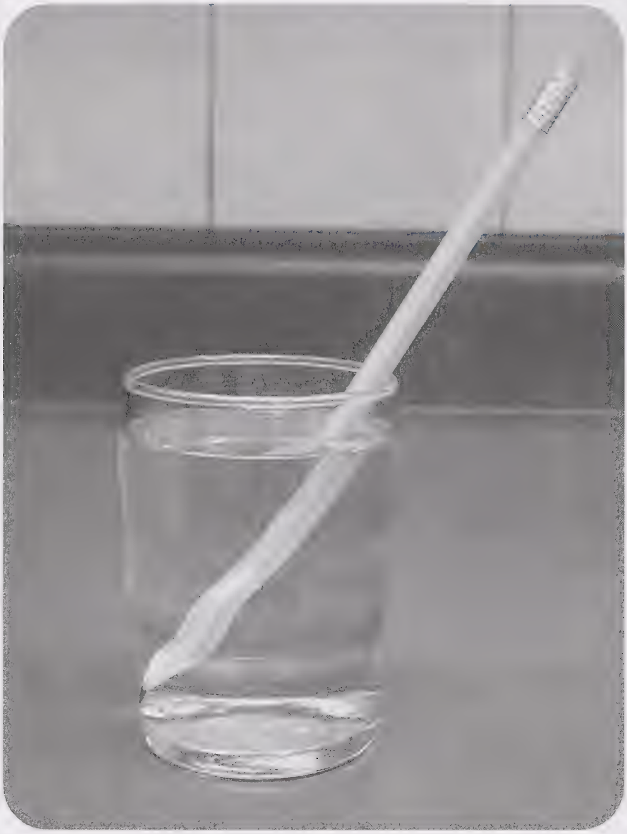
In this lesson you reviewed slides, flips, and turns by using them to make tessellations with various figures. You discovered why some figures tessellate and some do not. You learned how to create tessellating designs.



Turn to Assignment Booklet 7A and complete the Lesson 1 Assignment.

Keep Assignment Booklet 7A until you have completed the entire booklet.

Optical Illusions



Life is filled with illusions. A pencil in a glass of water appears to be bent because the light rays are bent when they enter and leave the glass. Vertical stripes on clothing make you appear slimmer than horizontal stripes do because the eye tends to follow the direction of the stripes. When a truck that is stopped beside your car starts moving forward, it seems like you are moving backward because you tend to view large objects as being stationary and smaller objects as moving.

In this lesson you will be introduced to optical illusions. You will see optical illusions that occur in nature and those that are created by people. You will view two-dimensional (2-D) and three-dimensional (3-D) optical illusions and investigate how they are made.

Activity 1

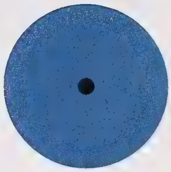
Today you will investigate several common optical illusions.


NASA

One of the most fascinating optical illusions can be seen just by looking up in the sky.

The full moon is a surprising natural illusion. When the full moon is just above the horizon, it appears to be much larger than when it is directly overhead. Tricked by this illusion, many photographers have been disappointed when the moon in their photographs seems much smaller than they thought it would.

1. You can do an experiment to experience an illusion similar to the full-moon illusion using the blue circle at the right. Hold the page at arm's length and stare at the black dot in the centre of the circle for about 30 s. Then look straight up and stare at the ceiling. Describe the image you see.



2. Repeat the experiment in question 1, but this time, after you stare at the circle, stare at a wall that is about 5 m to 10 m straight ahead of you. Describe the image you see.

3. How are the two images you saw in questions 1 and 2 similar to the full-moon illusion?

Check your answers on page 90 in the Appendix.

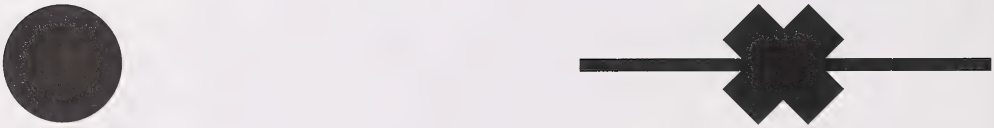
4. This optical illusion lets you experience a blind spot. Hold the following picture of the circle and cross directly in front of your face at arm's length.

   a. Close your left eye and stare at the circle with your right eye. Slowly bring the picture towards you. Describe what happens to the cross.

   b. Close your right eye and stare at the cross with your left eye. Slowly bring the picture towards you. Describe what happens to the circle.

5. Now do these experiments to learn about another feature of illusions with blind spots.

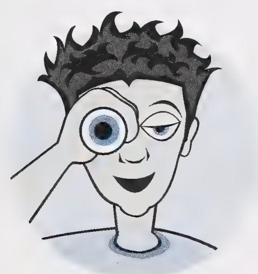
   a. Hold the following picture at arm's length. Repeat the procedure you followed in question 4.a. Describe what happens to the line at your blind spot.



b. Hold the following picture at arm's length. Repeat the procedure you followed in question 5.a. Describe what happens to the blue circle at your blind spot.

Check your answers on page 90 in the Appendix.

6. Have you ever seen through something solid? To do this, roll up a piece of paper to form a tight (approximately 2 cm to 3 cm in diameter), long tube (approximately 20 cm to 30 cm long).



Hold the paper tube in your right hand. Look through the tube with your right eye and focus on a distant object. (Keep both eyes open.)

Place the flat palm of your left hand a short distance in front of your face so it blocks the view of your left eye.

What is the surprising result?

7. An illusion you may be familiar with is a revolving barber pole. To imitate it, do the following:

- Cut out the striped rectangle from the Appendix.
- Form a cylinder with the rectangle by taping together the left and right edges of the rectangle with transparent tape. Be careful to match the coloured edges to form a continuous colour spiral. (You may find it helpful to wrap it around a paper-towel roll.)
- Now, hold the cylinder upright and turn it.

a. What appears to happen to the stripes when you turn the cylinder clockwise?

b. What appears to happen to the stripes when you turn the cylinder counterclockwise?

8. Place the stem of a stemmed glass over top of the following words.

CARBON DIOXIDE

a. Describe what you see.
b. Explain the illusion.

Check your answers on page 90 in the Appendix.

Activity 2

Today you will explore optical illusions involving two-dimensional geometric shapes.

In Activity 1 you saw some illusions that are part of the natural world. In this activity you will see how some illusions are created by drawing 2-D pictures.

1. **a.** Quickly read the phrase you see in the triangle in the picture shown above. Now, cover it and write down what you read. Compare the two phrases.

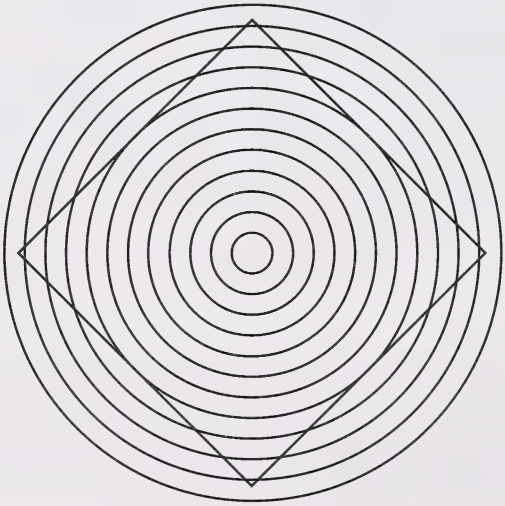
   b. Explain the illusion.

2. a. In the following picture, are the heavy lines parallel?

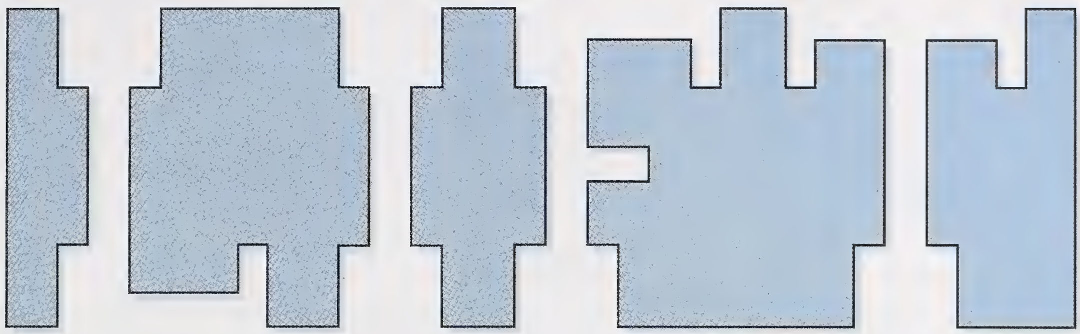
   b. Explain the illusion.

3. a. Look at the shape that appears among the set of circles and then describe that shape.

   b. Test your answer and explain the illusion.



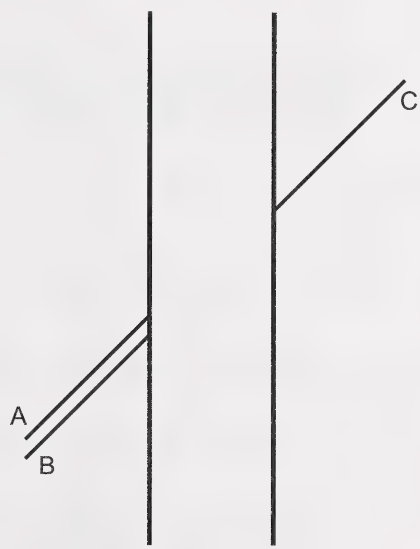
4. a. What do you see in the following picture?



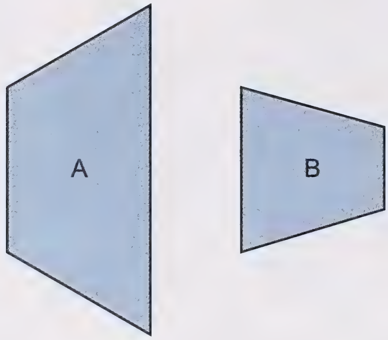
b. Explain the illusion.

5. a. In the following picture, does line C appear to be more like a continuation of line A or line B?

b. Test your answer and explain the illusion.

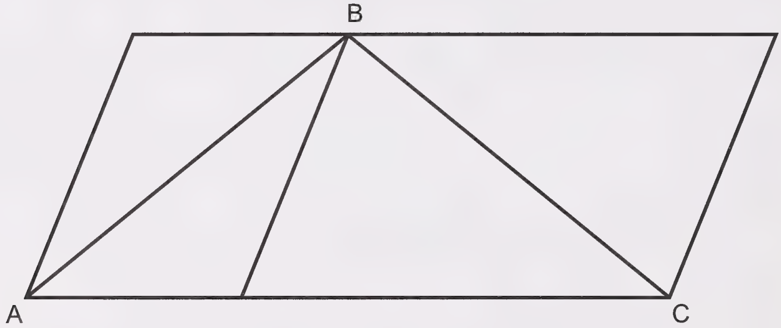


6. a. In the following picture, which appears longer, the left side of trapezoid A or the left side of trapezoid B?



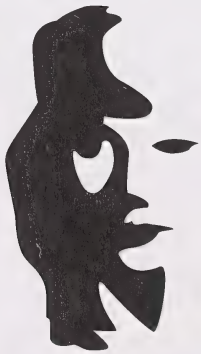
b. Test your answer and explain the illusion.

7. a. In the picture below, does the line segment from B to A appear to be the same length as the line segment from B to C?

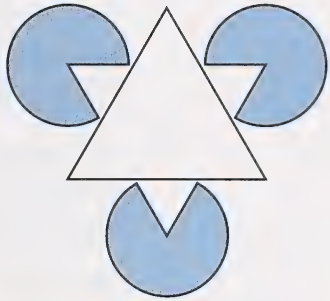


b. Test your answer and explain the illusion.

8. What different things do you see in the following picture?



9. Sometimes you think you see things that are not really there!



a. What four shapes have actually been drawn in the picture shown at the right?

b. Describe the illusion.

If you have access to the Internet, you can explore a variety of optical illusions at the following website:

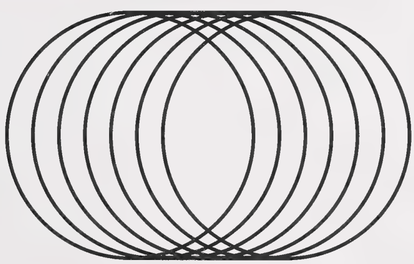
http://www.illusionworks.com

Check your answers on pages 90 to 92 in the Appendix.

Activity 3

Today you will see how some illusions are created by drawing 3-D pictures.

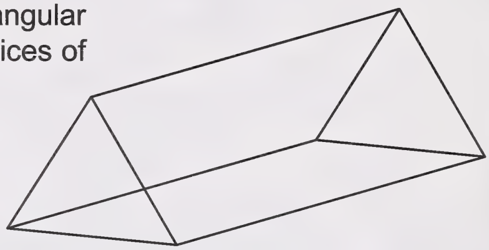
Which end of the cylinder appears closest to you?



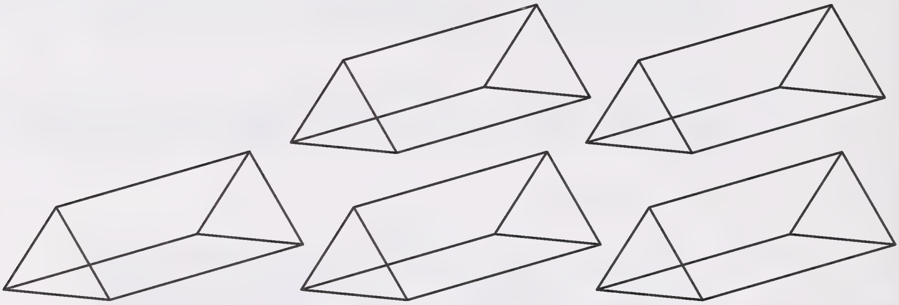
In Module 6 the 2-D pictures you drew of 3-D objects were illusions because they were drawn in a way that made them look 3-D.

1. Your eyes can play tricks on you when you draw 3-D objects. For example, in the picture of the cylinder shown on the previous page, is the opening of the cylinder on the right or the left?

2. The picture at the right shows a triangular prism drawn by connecting the vertices of two triangles.

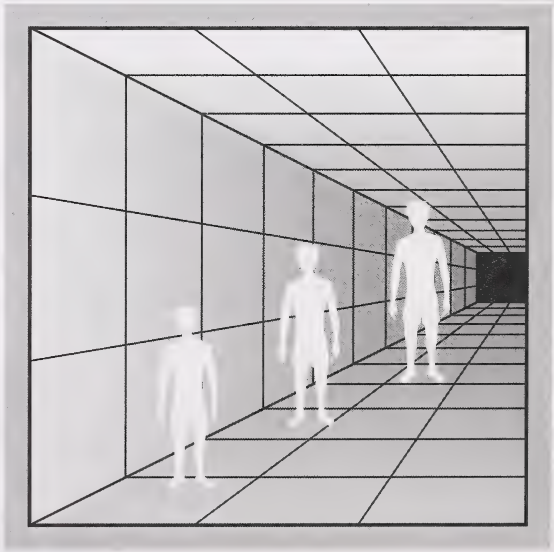


   Notice that when you stare at the prism and focus on different faces as being in the foreground, its orientation changes. To show this, shade in a different face in each of the following prisms.



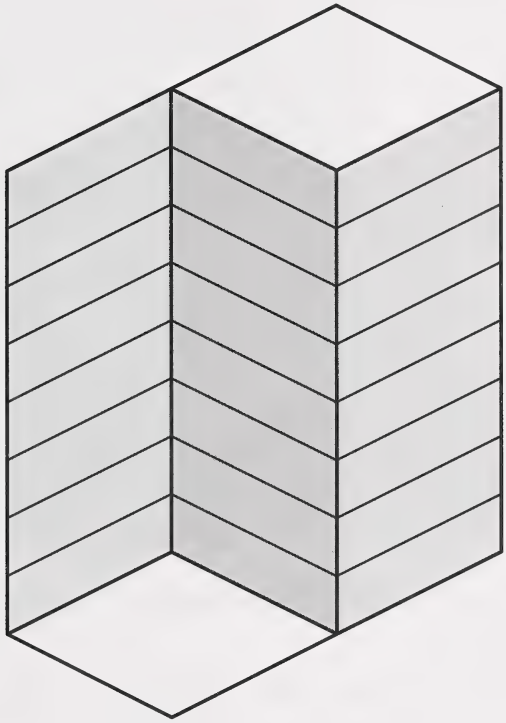
3. **a.** In the picture at the right, which person appears the largest?

   b. Test your answer and explain the illusion.

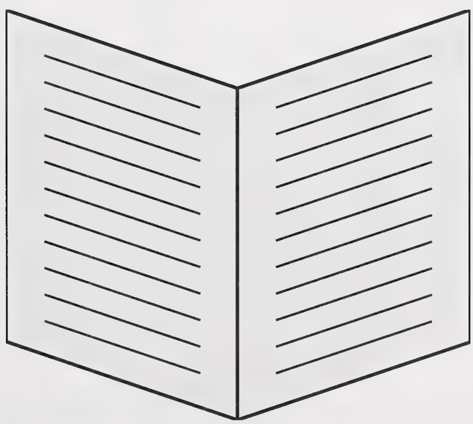


Check your answers on pages 92 and 93 in the Appendix.

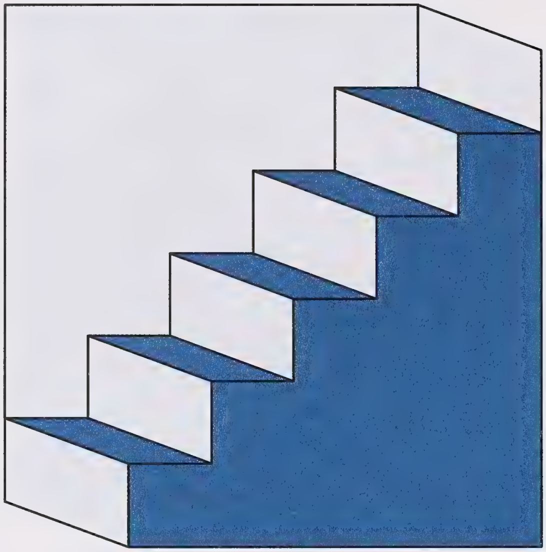
4. Describe the illusion in the following picture.



5. Is the book in the picture opening towards you or away from you? Explain.



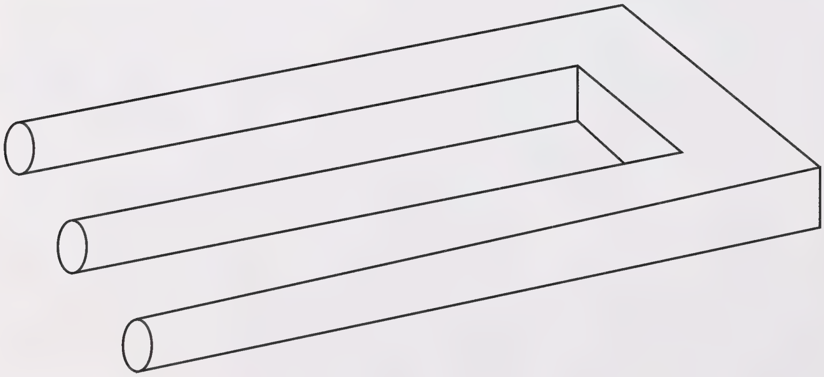
6. Look at the picture of the dark staircase.



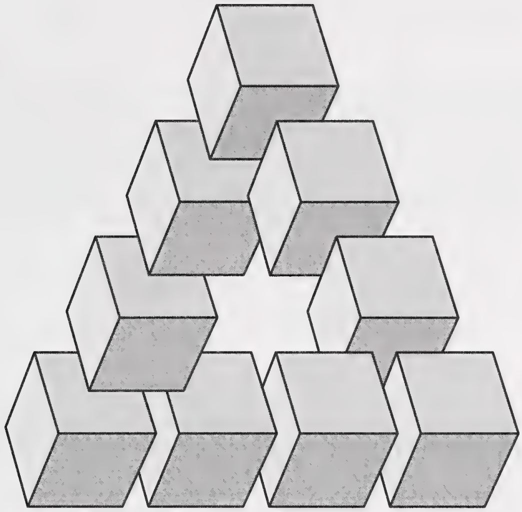
a. Going from left to right, what direction would the dark staircase take you?

b. Turn the picture 90° counterclockwise. Describe what you see.

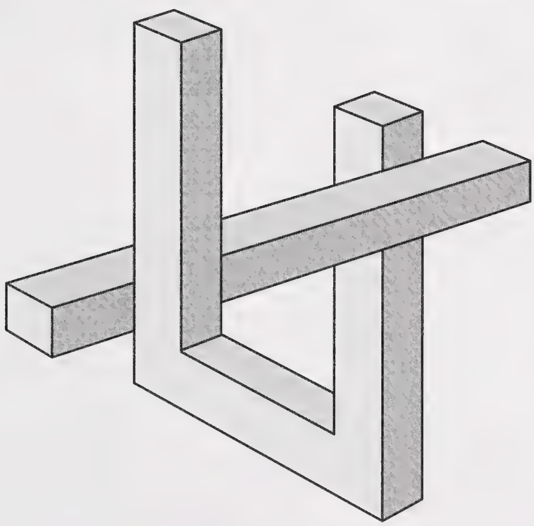
7. Explain why it would be impossible to actually make the 3-D object shown in the following picture.



8. Explain why it would be impossible to actually arrange cubes like those shown in the picture. (**Hint:** Try to build it using interlocking cubes.)



9. Explain why it would be impossible to actually build a structure like the one shown below.



Check your answers on pages 93 and 94 in the Appendix.

Sharing Time

Now it's time to share what you have been learning with your home instructor.

Access the following website:

http://dragon.uml.edu/psych/illusion.html

Discuss the illusions on this website with your home instructor. Which illusions are your favourites?

Challenge Activity

The following 14 objects are hidden in the picture on the next page. Find as many as you can. Use a highlighter to colour each object you find.

- teacup
- sailboat
- slipper
- toothbrush
- tack
- pyramid
- hockey stick
- spatula
- ice-cream cone
- pencil
- needle
- candle
- power drill
- turtle

Check your answers on page 95 in the Appendix.

Conclusion

In this lesson you were introduced to optical illusions. You saw optical illusions that occur in nature and those that are created by people. You viewed 2-D and 3-D optical illusions and investigated how they are made.

O. ALAMANY & E. VICENS/CORBIS

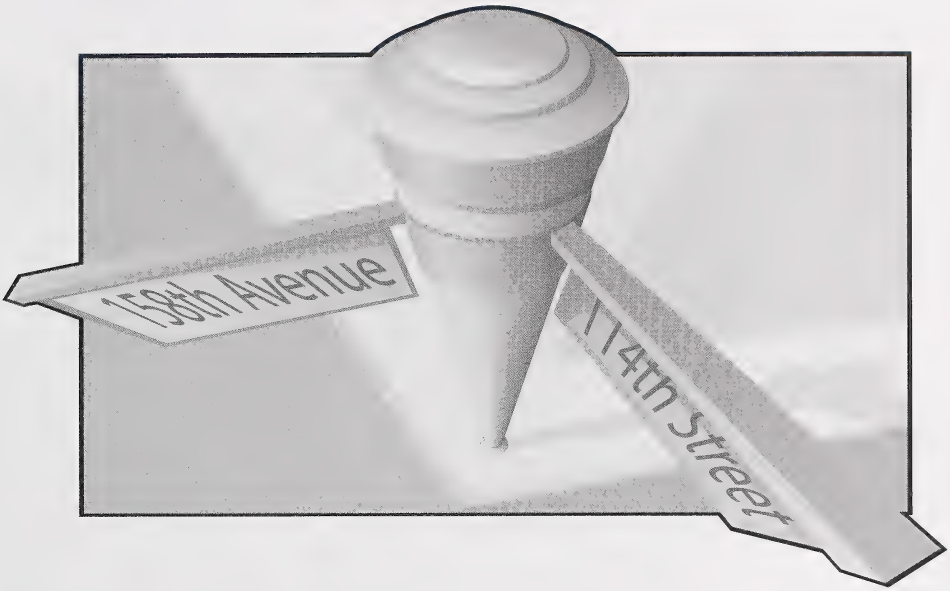
A very common optical illusion you likely have seen in summer is the appearance of "water" on the hot pavement or off in the distance across a stretch of bare land. As you approach the mirage, it gradually disappears. The "water" is really a bit of the sky that you see. Light coming from the sky is bent as it passes through the heated air rising from the pavement or ground.

Turn to Assignment Booklet 7A and complete the Lesson 2 Assignment.

When you are done, submit Assignment Booklet 7A to your teacher to be marked.

Lesson 3

Plotting Points



Most cities in Western Canada are built on a grid system using numbered streets and avenues. Armed with a map and this knowledge, you should be able to travel to any address, even in unfamiliar parts of the city.

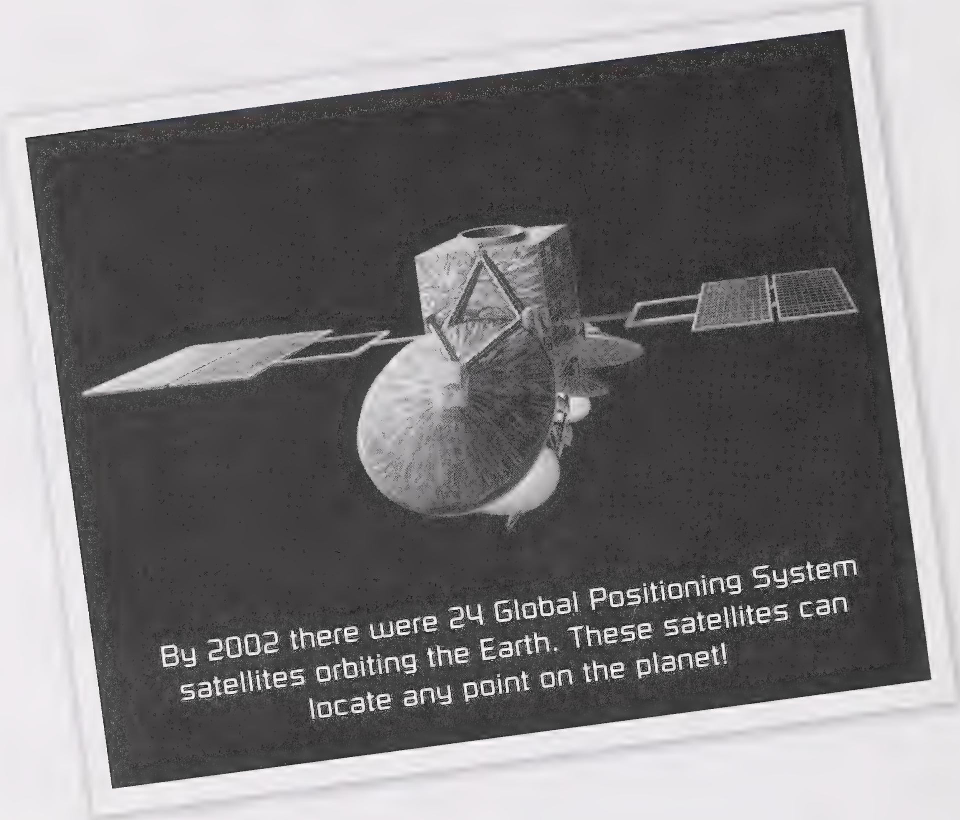
When the land on the Prairies was surveyed over a century ago, surveyors also used a grid system. This system, which is still in use today, was used by the early pioneers to locate their homesteads.

A global grid system uses longitude and latitude. This system, together with satellite technology, can be used with a high degree of precision to locate any point on Earth.

In this lesson you will review what you know about ordered pairs on a coordinate grid by using the global system of latitude and longitude. You will draw figures and pictures by using ordered pairs to plot points. You will investigate what happens when you slide, turn, or flip figures on a coordinate grid.

Activity 1

Today you will explore a grid system used to locate points on Earth.

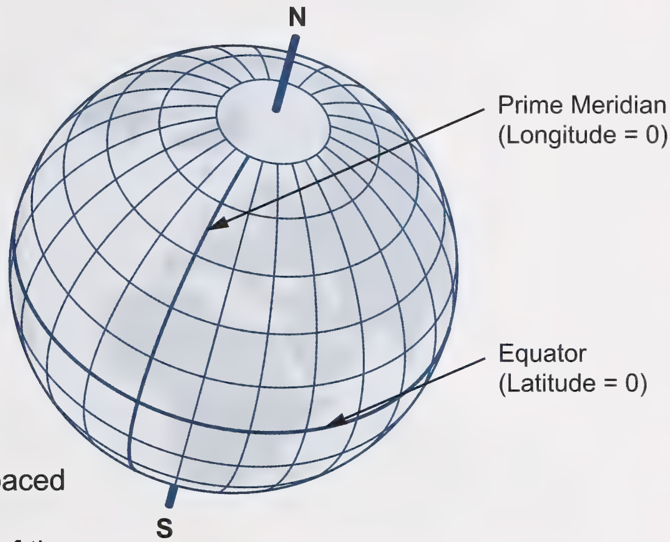


NASA

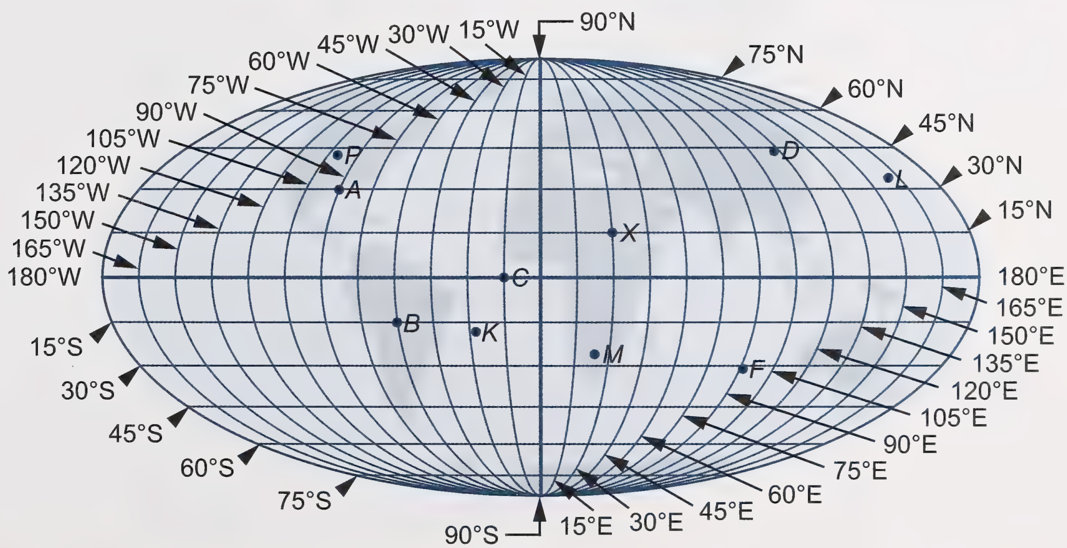
Twenty-four Global Positioning System (GPS) satellites orbit 20 000 km above the Earth. A GPS receiver will show you your exact location (an ordered pair of latitude and longitude coordinates) on an imaginary grid that covers the Earth. One important use of this technology is to locate the nearest police, fire, and medical services in an emergency.

The following picture shows this imaginary grid. The vertical arcs from the North Pole to the South Pole are the **lines of longitude**.

The prime meridian is at 0° longitude and passes through Greenwich, England. The other vertical arcs (spaced 15° apart) represent the locations east or west of the prime meridian. The parallel horizontal lines circling the Earth are the **lines of latitude**. The equator is at 0° latitude, and the other horizontal lines (spaced 15° apart) indicate the locations north or south of the equator.



There are 360° of longitude all the way around the globe. Halfway around the globe from the prime meridian is the International Date Line, where 180°E equals 180°W. If you cut open the globe down the International Date Line and flatten it out, it looks like this:

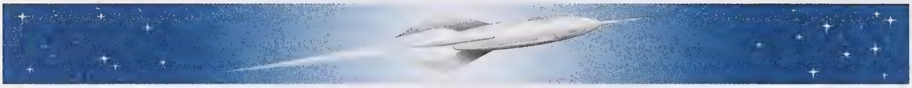


You can see how any place on Earth can be identified by its location on this grid.

Example

What are the coordinates of the location in Africa identified by the letter X on the map on the previous page?

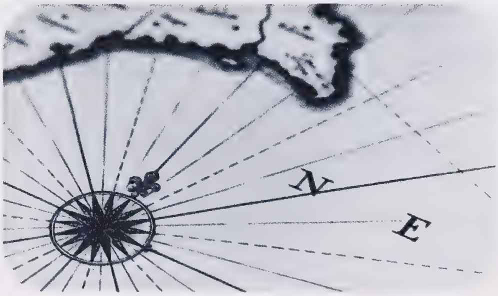
Location X is north of the equator. The horizontal line, parallel to the equator, that passes through location X is marked 15°N. The vertical line, running north and south, that passes through location X is marked 30°E. Location X is 30° east of the prime meridian. The coordinates of location X are Latitude: 15°N, Longitude: 30°E.



Use the map on the previous page to answer questions 1 to 9.

1. Copy and complete the following table in your notebook by writing the coordinates for each location.

Location	Latitude	Longitude
A		
B		
C		
D		
F		



2. Mark the following locations on the map.

Location	Latitude	Longitude
G	30°N	45°W
H	15°S	135°W
I	0°	15°E
J	60°N	135°E

3. Only some of the imaginary grid lines are drawn on the map. There are many locations between them. Copy and complete the following table in your notebook. Estimate the coordinates for latitude and longitude, in whole degrees, for each location listed in the table.

Location	Estimated Latitude	Estimated Longitude
K		
L		
M		
P		

4. Estimate and mark (approximately) the following locations on the map.

Location	Latitude	Longitude
Q	5°S	160°W
R	55°S	68°W
T	85°N	10°E
U	22°S	110°E

Check your answers on pages 96 and 97 in the Appendix.

5. Imagine drawing a quadrilateral with the least perimeter so that all of South America is inside it. List the coordinates of the corners.

6. If you were at location *C* on the map and you moved 45°N and 90°E, write the latitude and longitude of your new location.

7. In what directions and in how many degrees would you move to go from location *F* to location *A* on the map?

8. **a.** Describe all the ways you could go from location *C* to location *X* along the grid lines, always moving towards location *X*.

   b. Are the total distances for each of your different answers to question 8.a. always the same? Explain.

9. Use the map of Alberta shown at the right to estimate the coordinates for latitude and longitude, in whole degrees, for each of the following locations.

   a. Fort McMurray
   b. Edmonton
   c. Calgary

Check your answers on page 97 in the Appendix.

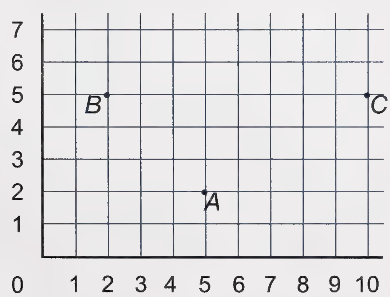
Activity 2

Today you will investigate what happens when figures slide on a coordinate grid.

First, study the following example to review how points are located on a coordinate grid.

Example

Determine the ordered pairs for points *A*, *B*, and *C* on the grid.

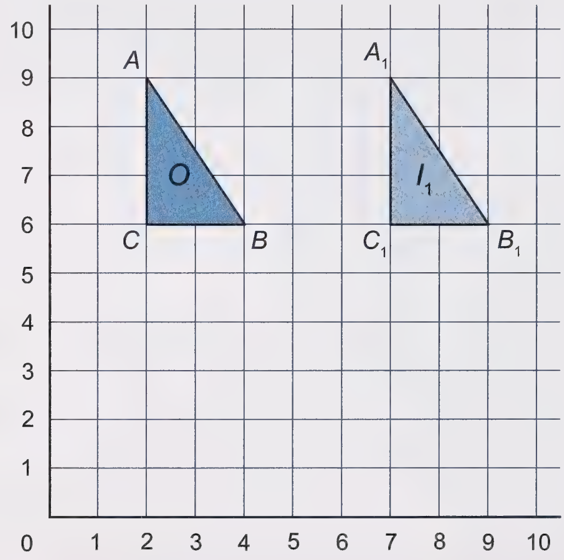


Remember, the first coordinate of the ordered pair tells you how far the point lies to the right of the vertical axis. The second coordinate of the ordered pair tells you how far the point lies up from the horizontal axis. Point *A* lies 5 units to the right and 2 units up. The coordinates for *A* are (5, 2).

Point *B* lies 2 units to the right and 5 units up. The coordinates for *B* are (2, 5).

Point *C* lies 10 units to the right and 5 units up. Point *C* is located at (10, 5).

Use the following grid to answer questions 1 to 7.



The dark blue triangle is the starting location for triangle *ABC*. It is called the original triangle, and so it is labelled with the letter *O*.

1. Copy and complete the following table by listing the coordinates for each vertex of the original triangle *ABC*.

Point	Coordinates
A	
B	
C	

Use tracing paper to trace triangle *ABC*. Cut it out and use it to answer questions 2 to 7.

2. Place the cutout on the grid so it fits on top of triangle *ABC*. Being careful not to turn the cutout, slide it to the right so it fits on top of the first slide image (labelled I_1, with vertices labelled A_1, B_1, and C_1). How many spaces to the right did it slide?

3. **a.** List the coordinates of each vertex of triangle I_1 by reading them from the scale.

   b. How can you use your answer from question 2 to answer question 3.a.?

Check your answers on page 98 in the Appendix.

4. Reposition the cutout on top of triangle *ABC* and slide it down 4 units.

   a. Draw the slide image on the grid. Label it I_2 and label its vertices A_2, B_2, and C_2.

   b. List the ordered pair for each vertex of triangle I_2.

5. **a.** Draw the slide image of triangle *ABC* on the grid as if the slides from question 2 and question 4 were combined (done one after the other instead of returning to the original location). Label the image I_3 and label its vertices A_3, B_3, and C_3.

   b. Describe the single straight path that slides the original triangle to triangle I_3.

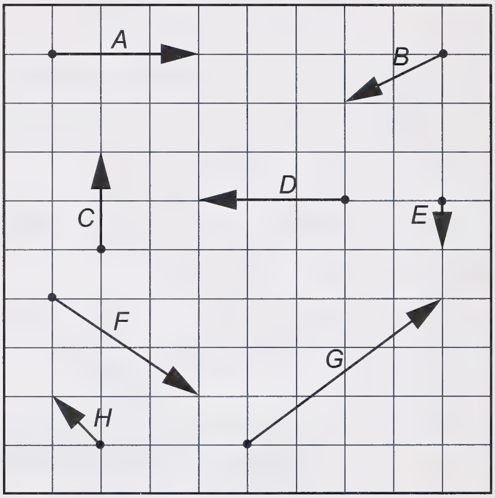
6. a. Predict the ordered pairs of the vertices of slide image I_4 if you slide the original triangle ABC 2 units to the left and 6 units down. Explain.

   b. Verify your prediction by sliding your cutout and drawing triangle $A_4B_4C_4$.

7. The ordered pair for vertex A_5 in slide image I_5 is (4, 10).

   a. Use your cutout to make the slides and draw slide image $A_5B_5C_5$.
   b. Write the ordered pairs for its other vertices.
   c. Describe the single straight path that slides triangle ABC to triangle I_5.

8. The distance and direction of slides on the coordinate plane are often represented with slide arrows. The slides for arrows *A* and *B* are described. Copy and complete the following table by describing the slides for arrows *C*, *D*, *E*, *F*, *G*, and *H*.



Slide Arrow	Description of Slide
A	Right 3
B	Left 2, Down 1
C	
D	
E	
F	
G	
H	

9. On a piece of grid paper from the Appendix, draw and label the following slide arrows.

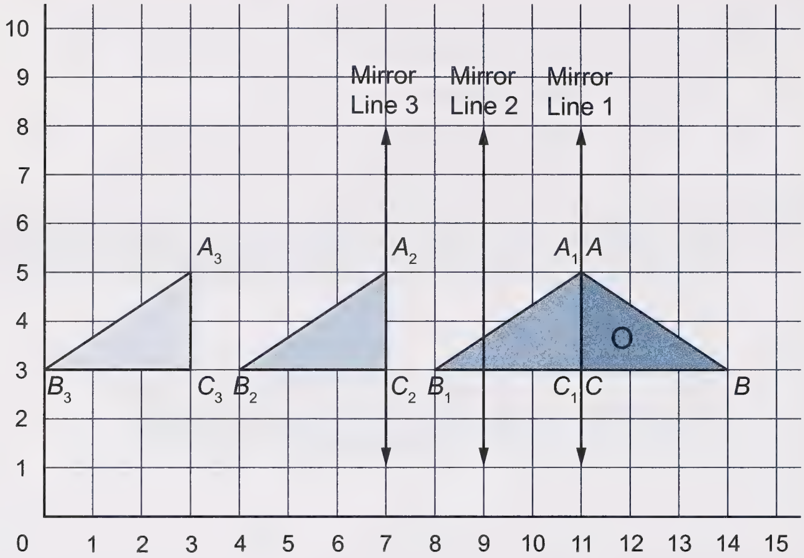
- *A*: (Up 3)
- *B*: (Right 4, Down 2)
- *C*: (Left 1, Up 6)
- *D*: (Right 5, Up 5)
- *E*: (Left 3, Down 4)

Check your answers on pages 98 to 101 in the Appendix.

A.3 Activity 3

Today you will investigate what happens when you flip or flip and slide figures on a coordinate grid.

Triangle *ABC* and three of its flip images are shown on the grid. Use the grid to answer questions 1 and 2.



1. Copy and complete the following table by reading the coordinates from the scale on the grid.

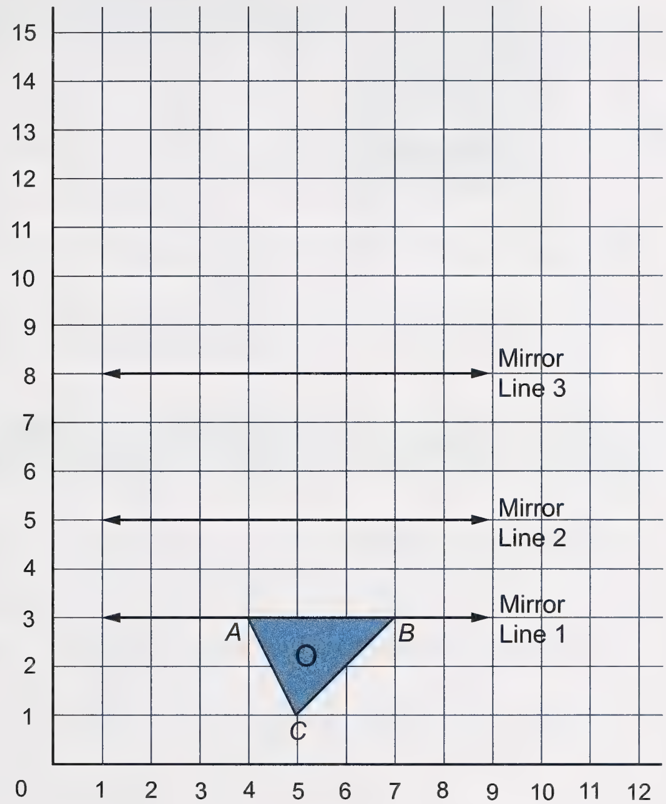
Triangle	*ABC*	$A_1B_1C_1$	$A_2B_2C_2$	$A_3B_3C_3$
Coordinates of Vertices	*A*:	A_1:	A_2:	A_3:
	B:	B_1:	B_2:	B_3:
	C:	C_1:	C_2:	C_3:

2. a. When a vertical mirror line is used to flip a figure, what pattern do you notice by comparing the second coordinates of the ordered pairs for each vertex of the original figure with those of its images? Explain why this happens.

   b. What motion moves triangle $A_1B_1C_1$ onto triangle $A_2B_2C_2$ and onto triangle $A_3B_3C_3$?

Check your answers on page 101 in the Appendix.

Triangle *DEF* and three mirror lines are shown on the following grid. Use the grid to answer questions 3 and 4.



3. a. Predict how the ordered pairs for the vertices of the images will be related to the ordered pairs for the vertices of the original figure.

b. Trace triangle *DEF* on tracing paper and cut it out. Use it to draw its flip image for each of the three mirror lines, and then label the vertices of each of the three images as follows.

Mirror Line	Image	Triangle
1	I_1	$D_1E_1F_1$
2	I_2	$D_2E_2F_2$
3	I_3	$D_3E_3F_3$

4. a. Copy and complete the following table in your notebook.

Triangle	*DEF*	$D_1E_1F_1$	$D_2E_2F_2$	$D_3E_3F_3$
Coordinates of Vertices	D:	D_1:	D_2:	D_3:
	E:	E_1:	E_2:	E_3:
	F:	F_1:	F_2:	F_3:

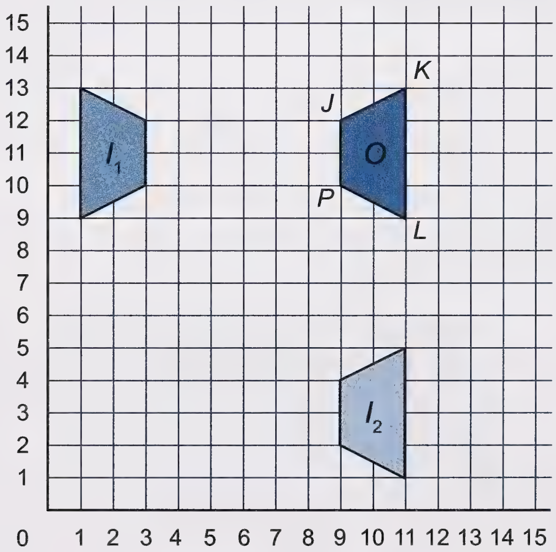
b. Verify your prediction from question 3.a. Explain the results.

c. What motion moves triangle $D_1E_1F_1$ onto triangle $D_2E_2F_2$ and onto triangle $D_3E_3F_3$?

Check your answers on page 102 in the Appendix.

5. On the following grid

- draw and label M_1 (the mirror line that makes flip image I_1) and M_2 (the mirror line that makes flip image I_2)
- label the vertices of flip image I_1 and flip image I_2



6. On a sheet of grid paper from the Appendix, draw and label the following quadrilaterals:

   - quadrilateral *ABCD* with *A* (2, 9), *B* (3, 8), *C* (3, 6), and *D* (1, 7)
   - quadrilateral *EFGH* with *E* (8, 1), *F* (7, 2), *G* (7, 4), and *H* (9, 3)

7. Describe how you can move quadrilateral *ABCD* onto quadrilateral *EFGH* by

   a. using only two flips (Draw and label both mirror lines.)

   b. using a combination of two flips and one horizontal slide (Draw the mirror lines and slide arrow.)

8. **a.** On a piece of grid paper from the Appendix, draw and label the following triangles:

   - triangle *ABC*, where *A* (2, 9), *B* (4, 6), and *C* (2, 6)
   - triangle *DEF*, where *D* (9, 2), *E* (6, 4), and *F* (6, 2)

   b. How are the coordinates of triangle *ABC* related to the coordinates of triangle *DEF*?

   c. Draw the mirror line that flips triangle *ABC* onto triangle *DEF*. (**Hint:** Use a Mira or fold the paper.)

9. Use the grid with triangle *ABC* and triangle *DEF* from question 8.a. If only the grid lines can be used as mirror lines, is it possible to move triangle *ABC* onto triangle *DEF* using a combination of flips and slides? Explain your answer.

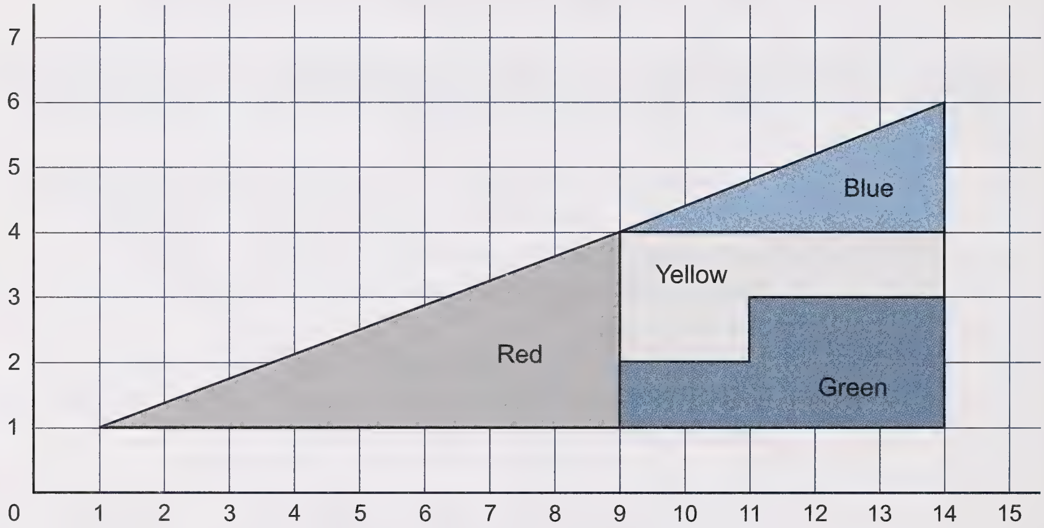
Check your answers on pages 103 to 105 in the Appendix.

Challenge Activity

1. Turn to page 127 in the Appendix and cut out the four coloured polygons.

   Begin by placing the cutouts on top of their matching polygons in the following grid. Next, slide the pieces as follows:

   - the red triangle (Right 5, Up 2)
   - the blue triangle (Left 8, Down 3)
   - the yellow hexagon (Left 3, Down 1)



2. What surprising thing happened? Explain why it happened.

Check your answers on page 106 in the Appendix.

Conclusion

In this lesson you reviewed what you know about ordered pairs on a coordinate grid by using the global system of latitude and longitude. You drew figures and pictures by using ordered pairs to plot points. You investigated what happens when you slide, turn, or flip figures on a coordinate grid.

Did you know that some drivers can, with the push of one button, transmit the GPS location of their vehicles to a service call center? With this location information, service reps can deliver alternative routing in the vehicle, hands-free, over the audio system, or even direct emergency providers to the location of the vehicle.

ONSTAR

Turn to Assignment Booklet 7B and complete the Lesson 3 Assignment.

Keep Assignment Booklet 7B until you have completed the entire booklet.

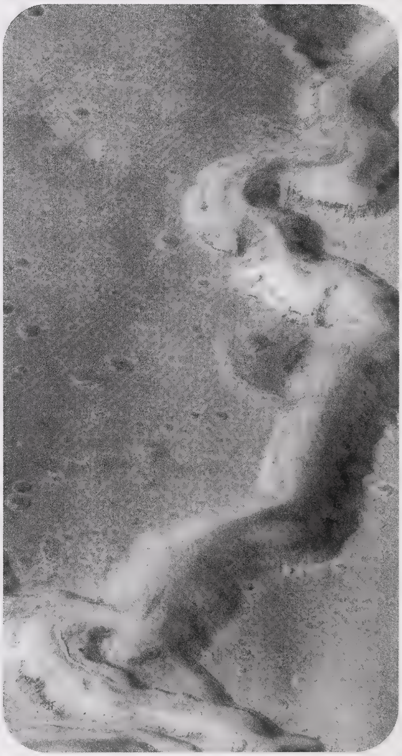
Module Summary

In this module you extended your knowledge of motion geometry and the coordinate plane. You investigated slides, flips, and turns by using them to make tessellations with various figures. You investigated why some figures tessellate and some do not. You learned how to create tessellating designs.

You were introduced to optical illusions. You saw optical illusions that occur in nature and others that are created by humans. You viewed 2-D and 3-D optical illusions and investigated how they are made.

You extended what you know about ordered pairs on a coordinate grid by using the global system of latitude and longitude. You drew figures and pictures by using ordered pairs to plot points. You investigated what happens when you slide, turn, or flip figures on a coordinate grid.

The examination of geometrical patterns can tell observers a great deal about the universe around them. For example, a close examination of the surface of Mars reveals valleys with shapes similar to those found on Earth. These valleys could only have been formed by flowing water. Mars at one time must have had oceans and rivers!



NASA

Turn to Assignment Booklet 7B and complete the Numbers in the News project.

When you are done, submit Assignment Booklet 7B to your teacher to be marked.

Keystrokes

Take out your calculator and complete the following exercises. They will help you review some of the ideas you have learned in Module 7.

Funky Feature: Calculation Illusion

Use your calculator to find each of the following products. Read the display and then turn your calculator upside down and read the display again.

1. Copy and complete the following tables in your notebook.

a.

Keystrokes	Product Displayed	Upside-down Display
76 × 118 =		
19 × 472 =		
59 × 152 =		
38 × 236 =		

b.

Keystrokes	Product Displayed	Upside-down Display
57 × 158 =		
38 × 237 =		
79 × 114 =		
19 × 474 =		

c.

Keystrokes	Product Displayed	Upside-down Display
19 × 99 =		
11 × 171 =		
33 × 57 =		
9 × 209 =		

d.

Keystrokes	Product Displayed	Upside-down Display
22 × 364 =		
14 × 572 =		
56 × 143 =		
44 × 182 =		

2. Explain why the products look the same upside down.

3. State a plan you can use to find the different factor pairs for each of the four products in question 1.

4. Use your plan and write at least one more pair of factors (not found in the table above, and other than 1 times itself) for each of the four products in question 1.

5. Try to find more factor pairs whose products look the same on the display both right side up and upside down.

Check your answers on pages 106 to 108 in the Appendix.

Funky Feature: Operations Gymnastics

6. For this activity, you may use **only** the keys 4, +, −, ×, ÷, and = to make your calculator display the number 7.

- You may use some or all of the keys.
- You may use the keys in any order.
- You may use the keys as many times as you wish.

Use tables like the following one to record your keystrokes for each solution you find. Try to find a solution that uses the fewest possible number of keystrokes.

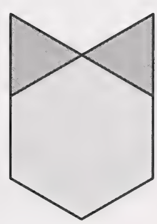
Keystrokes											
Display											

Check your answers on pages 108 and 109 in the Appendix.

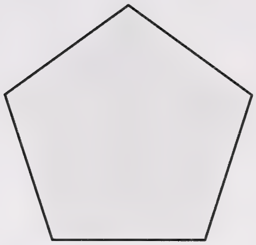
Review

The activities in this Review will help you review and apply what you learned in Module 7 and prepare for the final test. Discuss with your home instructor when you should begin the Review and how much of the Review you should complete.

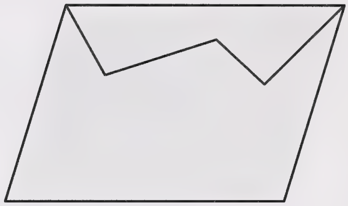
1. Karen used pattern blocks to make a kitten design for wrapping paper. Use pattern blocks to see if Karen's design will tessellate by using only slides. Draw a picture and use it to explain your answer.



2. Use tracing paper to copy the pentagon shown at the right. Glue it on cardboard and cut it out. Which pattern block could you use to fill in the gaps made when you try to tessellate the pentagon? Draw a picture and use it to explain your answer.



3. Use tracing paper to copy the parallelogram shown at the right. Glue it on cardboard and cut it out. Cut out the top part and slide it the bottom. Join and tape the edges to make a new shape.



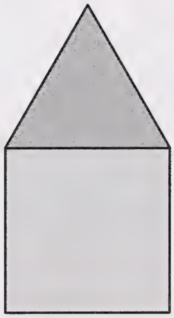
   a. Predict whether or not your new shape will tessellate. Explain your answer.

   b. Draw a picture and use it to justify your answer to question 3.a.

Check your answers on pages 109 and 110 in the Appendix.

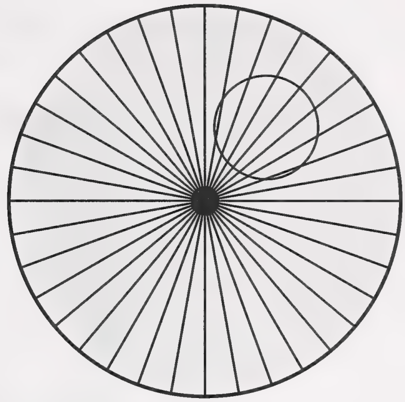
If you need help with questions 1 to 3, look back at Lesson 1, where you learned about tessellations. If you feel you need more practice, do question 4.

4. Joe used pattern blocks to make the house design shown. Use pattern blocks to see if Joe's design will tessellate by using only slides. Draw a picture and use it to explain your answer.



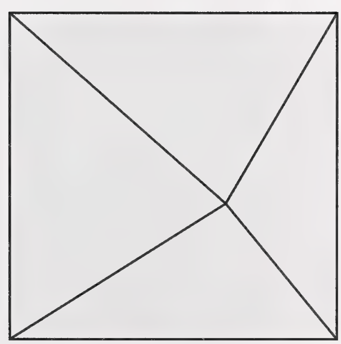
Check your answers on page 110 in the Appendix.

5. In the picture shown at the right, how do the spokes of the wheel affect the appearance of the smaller circle?

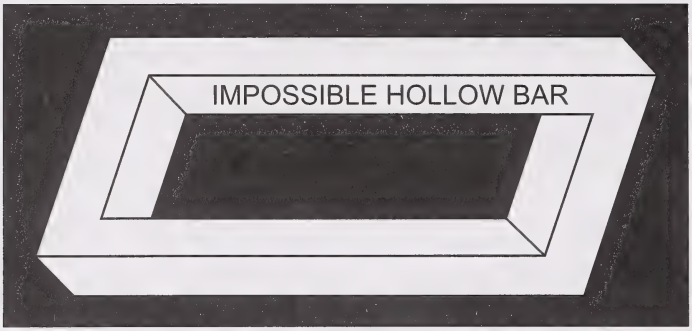


6. What different ways are there of looking at the picture shown at the right?

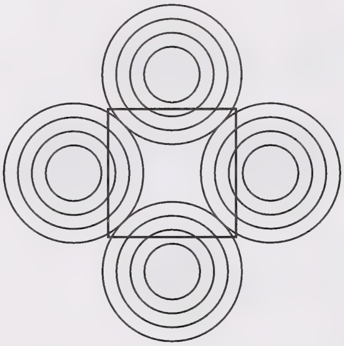
7. Explain the two different ways of viewing the pyramid.



8. Explain why it would be impossible to actually make the 3-D object shown in the picture below.



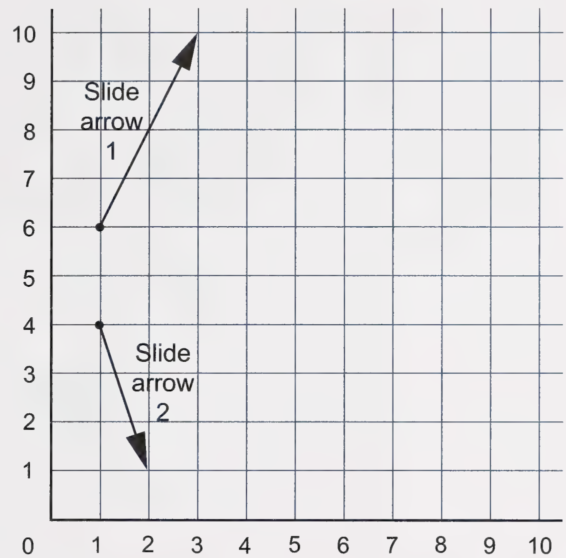
If you need help with questions 4 to 8, look back at Lesson 2, where you learned about optical illusions. If you feel you need more practice, do question 9.



9. **a.** What is the intended illusion?
b. What figures are actually drawn?
c. What causes the illusion?

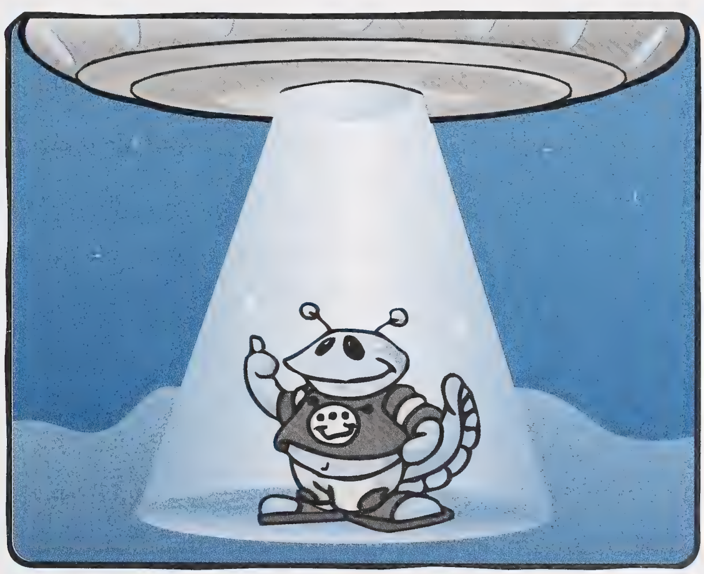
Check your answers on page 111 in the Appendix.

10. a. Use the following grid to draw and label quadrilateral *ABCD* with *A* (2, 5), *B* (5, 5), *C* (4, 4), and *D* (4, 3).

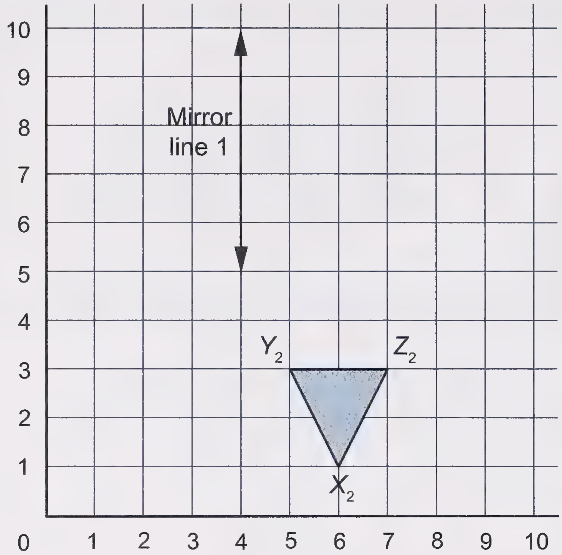


b. Slide *ABCD* the direction and amount shown by slide arrow 1. Draw and label image $A_1B_1C_1D_1$.

c. Slide $A_1B_1C_1D_1$ the direction and amount shown by slide arrow 2. Draw and label image $A_2B_2C_2D_2$.



11. a. Use the following grid to draw and label triangle *XYZ* where *X* (2, 9), *Y* (3, 7), and *Z* (1, 7).



b. Flip *XYZ* using mirror line 1. Draw and label image $X_1Y_1Z_1$.

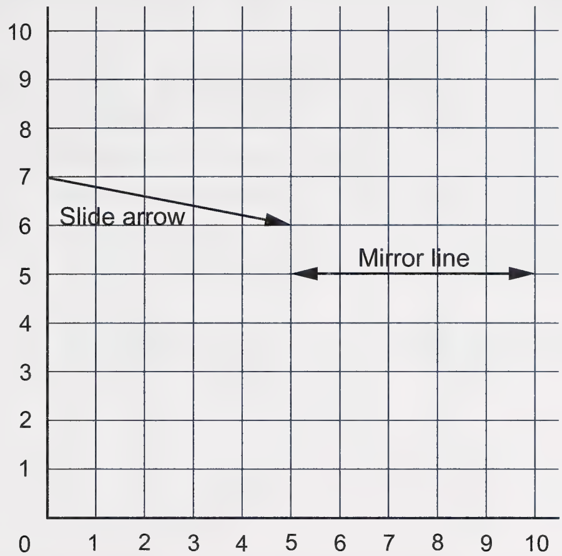
c. Draw and label the mirror line that flips $X_1Y_1Z_1$ to make image $X_2Y_2Z_2$.

If you need help with questions 10 and 11, look back at Lesson 3, where you learned about slides and flips on the coordinate plane. If you feel you need more practice, do question 12.

12. a. Use the following grid to draw and label triangle *MNP* where *M* (1, 9), *N* (4, 9), and *P* (3, 8).

b. Slide *MNP* the direction and amount shown by the slide arrow. Draw and label image $M_1N_1P_1$.

c. Flip $M_1N_1P_1$ using the mirror line. Draw and label image $M_2N_2P_2$.



Check your answers on pages 111 and 112 in the Appendix.

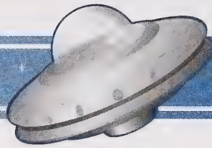
If you need additional work to master the material in this module, work through the following lessons on the Mathematics 6 Companion CD.

- Lesson 17: Slides and Flips
- Lesson 18: Using Ordered Pairs

After each lesson, you can print out an activity by clicking on the Activity button at the bottom of the screen.

Ask your home instructor to print out the solutions to the questions in each activity by clicking on the Parent Notes button at the bottom of the screen. Discuss your answers with your home instructor.

Just the Facts



Ask your home instructor to time you as you complete the following timed drill. Your goal is to complete all 25 questions in two minutes. At the end of two minutes, count how many questions you were able to complete. Then use the Answer Key in the Appendix to mark the drill, and record your score in the space provided. Before you move on, go back and complete any questions you did not finish.

Multiplication and Division Facts

$1 \times 4 =$	$63 \div 9 =$	$\begin{array}{r} 2 \\ \times 4 \\ \hline \end{array}$	$4\overline{)12}$	$1 \times 4 =$
$20 \div 5 =$	$\begin{array}{r} 0 \\ \times 4 \\ \hline \end{array}$	$1\overline{)3}$	$2 \times 4 =$	$10 \div 2 =$
$\begin{array}{r} 6 \\ \times 0 \\ \hline \end{array}$	$8\overline{)32}$	$5 \times 4 =$	$6 \div 6 =$	$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$
$6\overline{)12}$	$2 \times 1 =$	$6 \div 1 =$	$\begin{array}{r} 3 \\ \times 9 \\ \hline \end{array}$	$7\overline{)0}$
$5 \times 8 =$	$24 \div 3 =$	$\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$	$7\overline{)63}$	$3 \times 6 =$

Multiplication and Division Facts

Number completed in 2 minutes: ______
Number correct in 2 minutes: ______

Record your score on the Just the Facts Progress Chart.

Mental Math

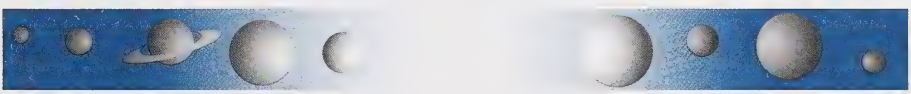
Is the best estimate always the one closest to the exact answer? No, not necessarily. It depends on the reason for making the estimate.

Example 1

Joni is mailing information to the 75 members of her ski club, and each letter requires a 48¢ stamp. What will the total cost of postage be?

In this case, Joni may choose to **overestimate** because she will have to pay GST when she purchases the stamps.

If you round each number up to a multiple of ten, you get $80 \times 50 = 4000$, so the cost is about \$40. Notice that because both numbers were rounded up, the estimate will be greater than the actual answer of 3600 (or \$36). **Note:** The actual cost, with GST included, is \$38.52.



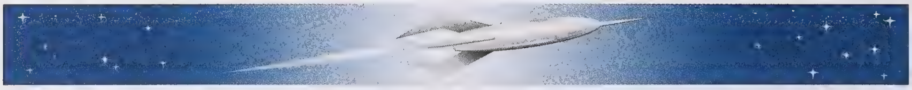
Example 2

Phil is buying a stamp album. He found one that has 75 pages, and each page holds about 48 stamps. How many stamps in total will the album hold?

Phil may wish to **underestimate** to be sure that the album will hold at least that particular number of stamps.

Using front-end rounding, you get $70 \times 40 = 2800$. Notice that because both numbers were rounded down, the estimate will be less than the actual answer of 3600.

Particularly appropriate for Example 2, you can **compensate** by rounding one number being multiplied up and the other number down. Since 75 is as close to 70 as it is to 80, round 75 down to 70 and round 48 up to 50 to get $70 \times 50 = 3500$ stamps, which is closer to the exact answer than the previous estimate is.



When multiplying two numbers, remember the following:

- Increasing both factors always gives a product greater than the actual product.
- Decreasing both factors always gives a product less than the actual product.
- Increasing one factor and decreasing the other factor gives a product between the greatest and least products.

Keep this in mind and remember to choose the strategy that is most useful to you. Try to practise this strategy whenever it is appropriate to use it.

Mathematics 6

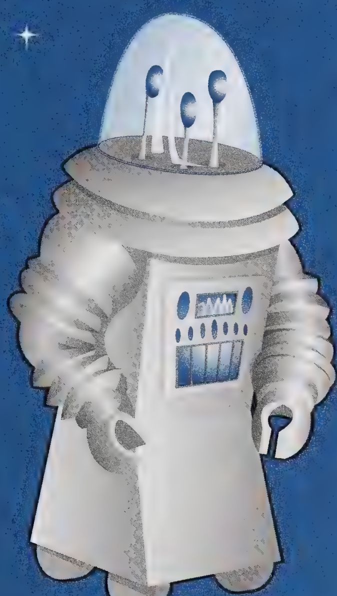
Appendix

Glossary

Answer Key

Image Credits

Learning Aids



Glossary

flip: to form a mirror image of a shape

line of symmetry: a line drawn through the middle of a geometric shape about which each half of the shape can be reflected onto the other half

lines of latitude: imaginary lines circling Earth and running parallel to the equator

lines of longitude: imaginary lines circling Earth and passing through the poles

mirror line: a straight line in which a geometric shape is reflected

slide: to move a geometric shape in a straight line without twisting or turning it

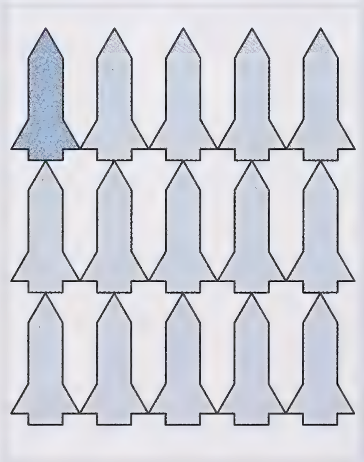
tessellate: to cover a surface using a geometric shape without gaps or overlaps

Answer Key

Lesson 1: Motion Geometry

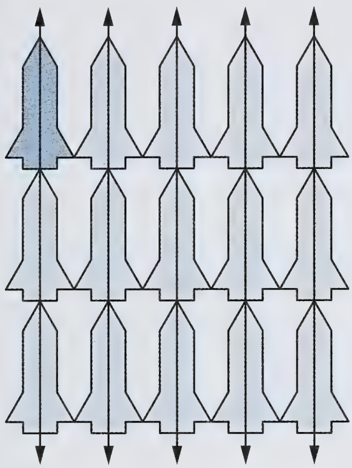
Activity 1

1. About three rows of five figures each can be traced across a sheet of paper.

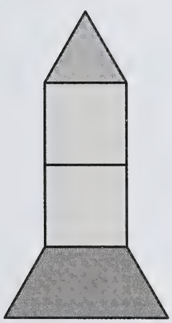


2. a. The gaps are the same shape as the figures you traced.

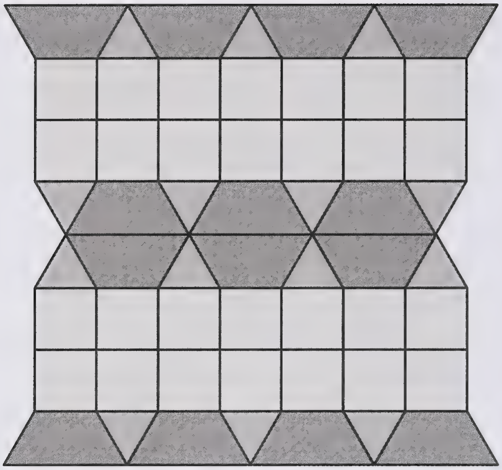
   b. You have to flip the cardboard figure or turn it 180° ($\frac{1}{2}$-turn) to make it fit in the gaps.

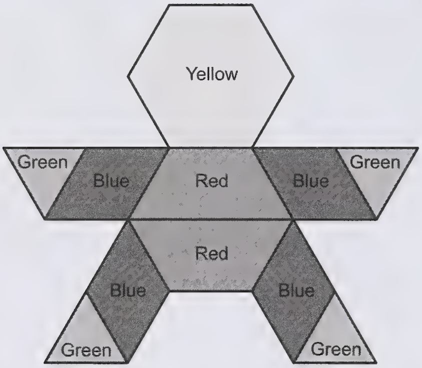
3. a. 

   b. The lines of symmetry are parallel to each other.

4. 

5. You can use slides only with the squares, but you have to flip the triangles and trapezoids to make a tessellation with the shuttle design from question 4.

6. 

7. a. 

b. • She could slide the triangle used for the left hand to make the right hand.

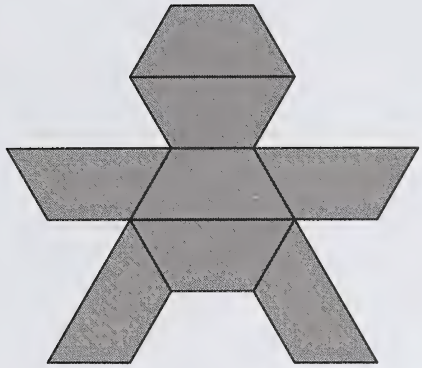
• She had to flip the triangles used for the hands to make the feet.

• She had to flip the rhombus used for the left arm to make the right arm.

• She had to turn the rhombuses used for the arms to make the legs.

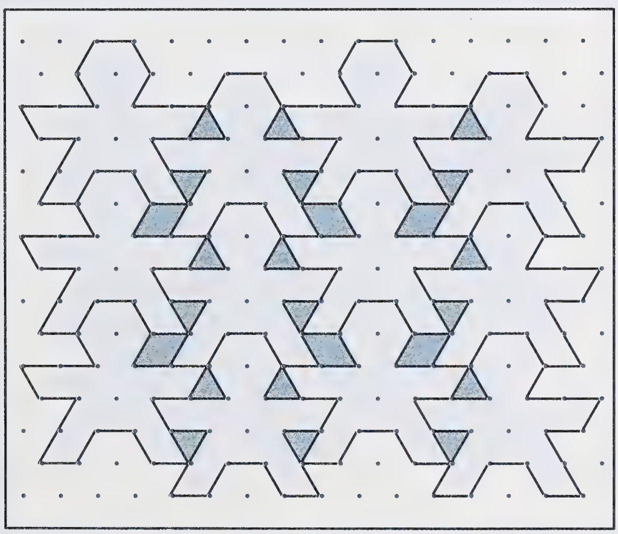
• She had to flip the trapezoid used for the top half of the body to make the bottom half of the body.

8.



- The top of the head slides to make the top of the body.
- The bottom of the head slides to make the bottom of the body and the arms.
- The trapezoids have to be turned to make the legs.

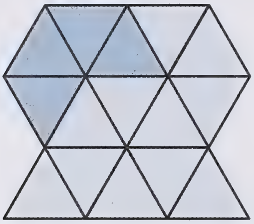
9. The figure does not tessellate. It leaves gaps that could be filled with green triangles and blue rhombuses.



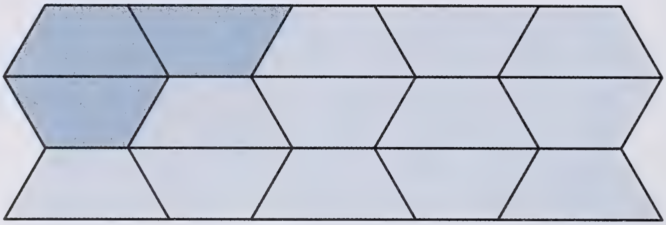
Activity 2

1. **a.** Each angle in a regular hexagon has a measure of 120°. 360 ÷ 3 = 120°
   b. The total measure of the three hexagon angles joined at each vertex is 360°.

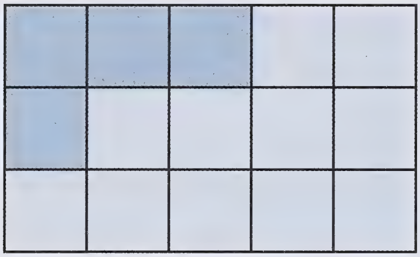
2. a. green triangle



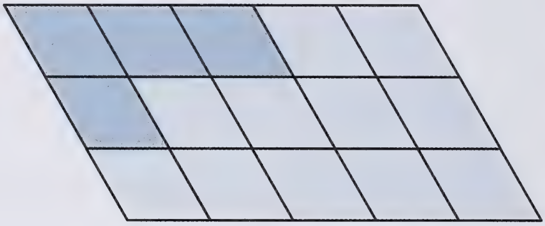
b. red trapezoid



c. orange square

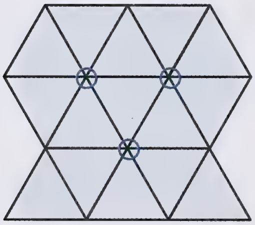


d. blue rhombus

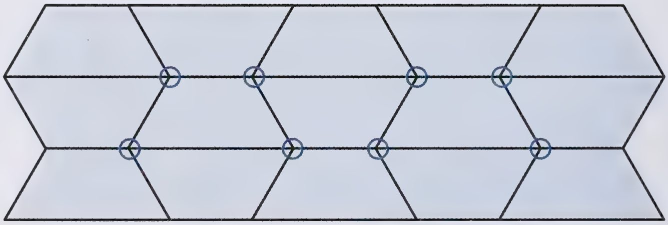


e. tan rhombus

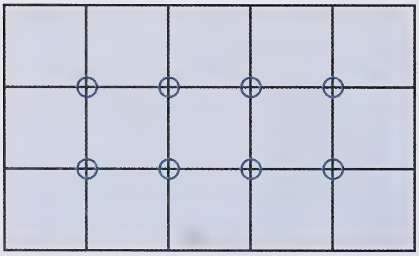
3. a. green triangle



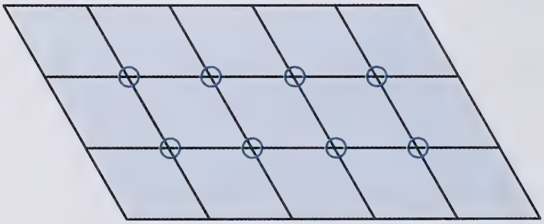
b. red trapezoid



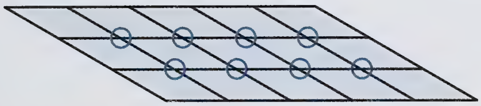
c. orange square



d. blue rhombus



e. tan rhombus



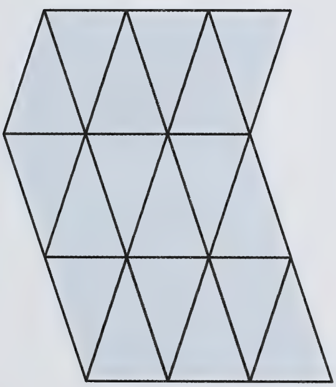
4.

Pattern Blocks	Sum of the Angles at Each Vertex	Is the Polygon Regular?
Yellow Hexagons	$120° + 120° + 120° = 360°$	yes
Green Triangles	$60° + 60° + 60° + 60° + 60° + 60° = 360°$	yes
Red Trapezoids	$60° + 60° + 120° + 120° = 360°$	no
Orange Squares	$90° + 90° + 90° + 90° = 360°$	yes
Blue Rhombuses	$60° + 60° + 120° + 120° = 360°$	no
Tan Rhombuses	$30° + 30° + 150° + 150° = 360°$	no

5. No, a polygon does not have to be regular in order to tessellate. The trapezoid and the rhombuses are not regular, but they tessellate.

6. a. The isosceles triangle will tessellate because you can put two of them together to form a parallelogram, and parallelograms tessellate.

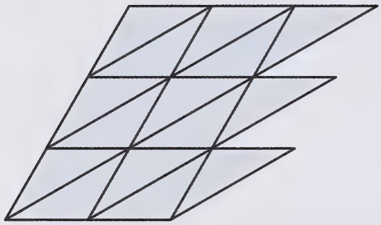
b.



The picture at the left shows that the isosceles triangle does tessellate.

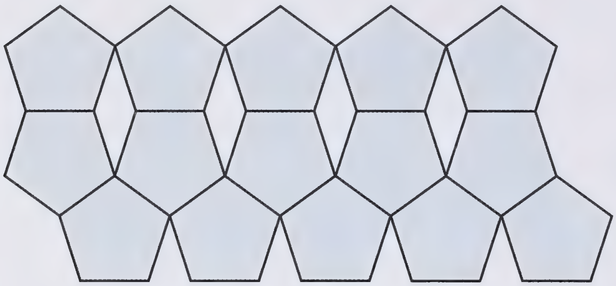
7. a. The scalene triangle will tessellate because, by using a half-turn, you can put two of them together to form a parallelogram, and parallelograms tesselate.

b.

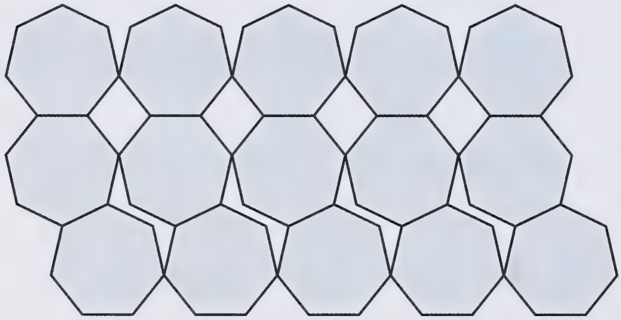


The picture at the left shows that the scalene triangle does tessellate.

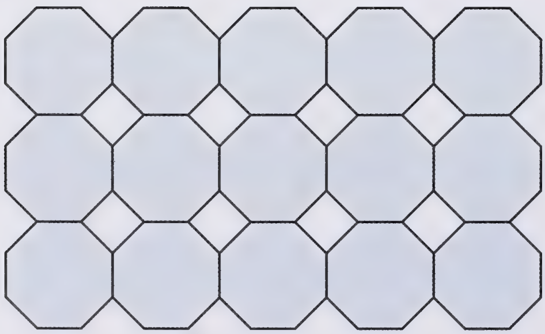
8. a. pentagon



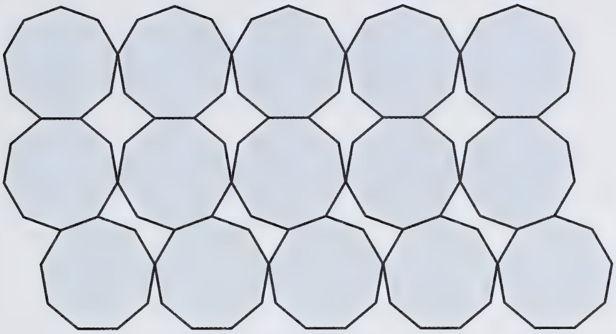
b. heptagon



c. octagon



d. nonagon



9. a.

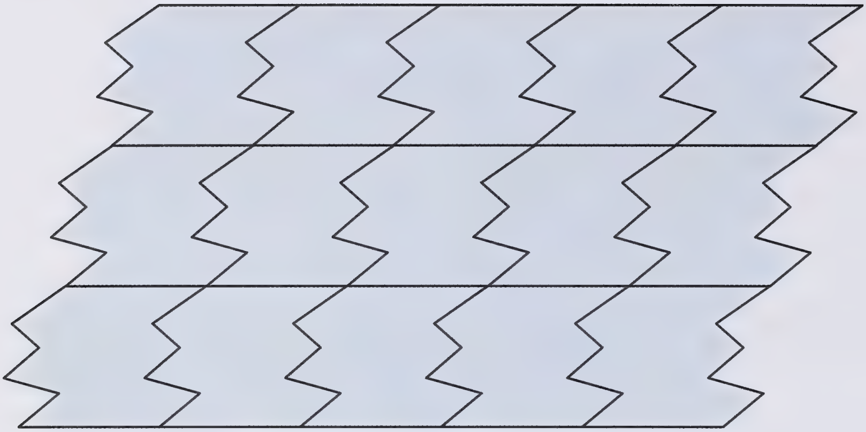
Regular Polygon	Size of Each Angle	Does It Tessellate?	Is the Size of Each Angle a Factor of 360°?
Triangle	60°	yes	yes ($6 \times 60° = 360°$)
Square	90°	yes	yes ($4 \times 90° = 360°$)
Pentagon	108°	no	no
Hexagon	120°	yes	yes ($3 \times 120° = 360°$)
Heptagon	almost 129°	no	no
Octagon	135°	no	no
Nonagon	140°	no	no

b. For a regular polygon to tessellate, the size of its angles must be a factor of 360°. Because their angles are all equal, there is only one way to fit them together.

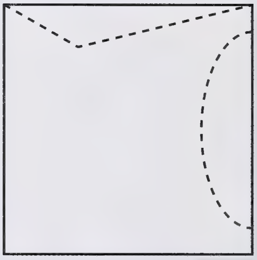
Activity 3

1. You can make a tessellation using slides. When you trace the shape, the design that sticks out on the left side will fit into the matching gap on the right side.

2.

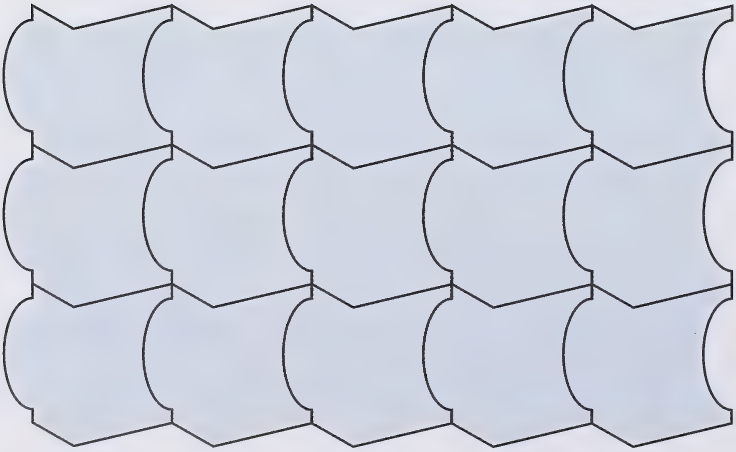


3. To make the shape, Lei first cut a curved piece out of the right side and slid it to the left side. She then cut a triangular piece out of the top side and slid it to the bottom side.



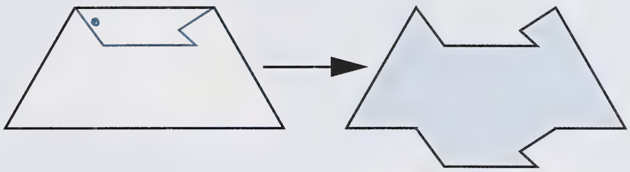
4. You can make a tessellation with Lei's shape using slides. When you trace the shape, you can fit the curved bump on the left side into the curved hole on the right side, and you can fit the triangular bump on the bottom side into the triangular hole on the top side.

5.

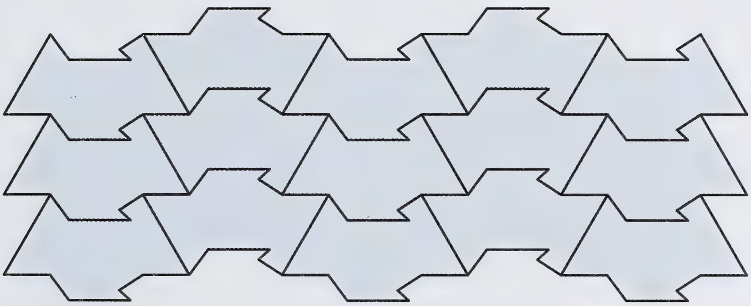


6. a. The top side of the trapezoid is parallel to the bottom side.

   b. Answers will vary. A sample tessellating shape is shown.



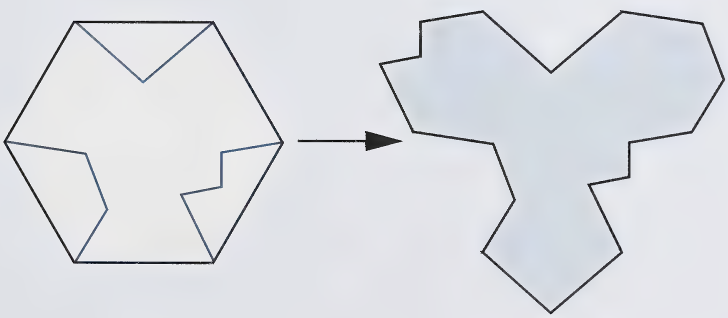
   c. Answers will vary. A sample tessellation is shown.

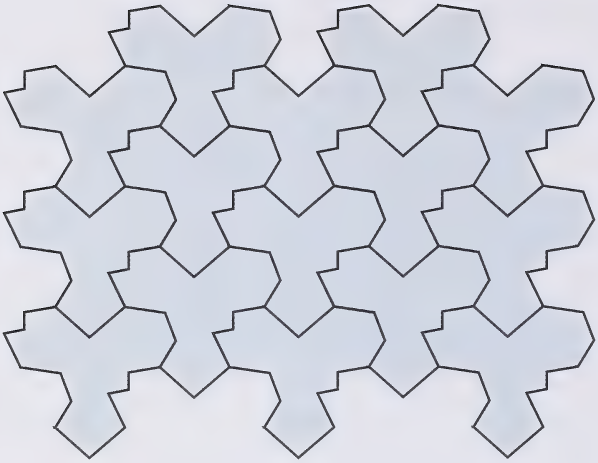


   d. You used slides and flips to make your tessellation.

7. a. The regular hexagon has three pairs of parallel sides.

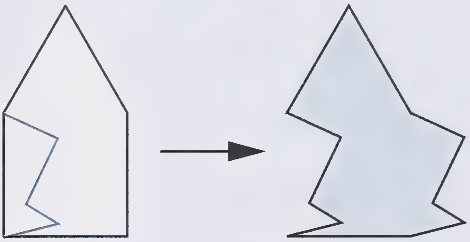
   b. Answers will vary. A sample tessellating shape is shown.

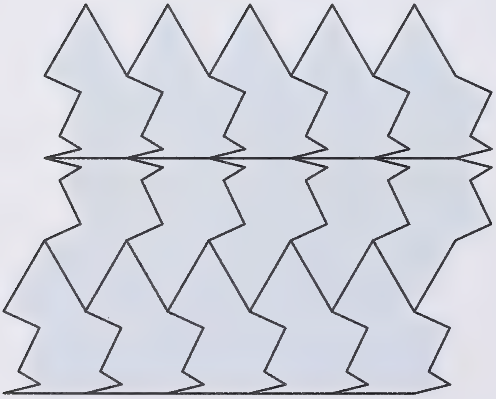


c. 

8. a. You cannot use the parallel side method you used in question 7 to make a tessellating shape with a regular pentagon because it has no parallel sides.

b. Answers will vary. A sample tessellating shape made with a pentagon (with one pair of parallel sides) is shown.



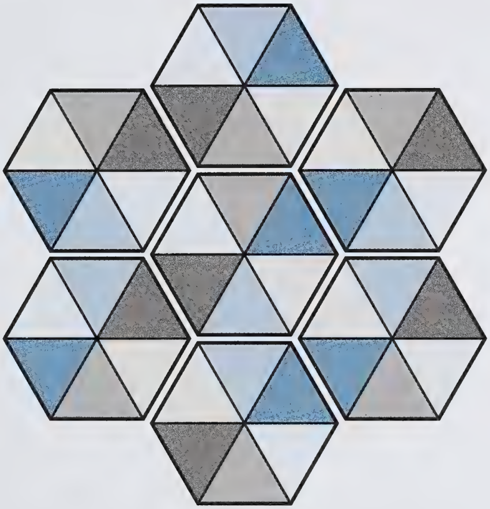
c. 

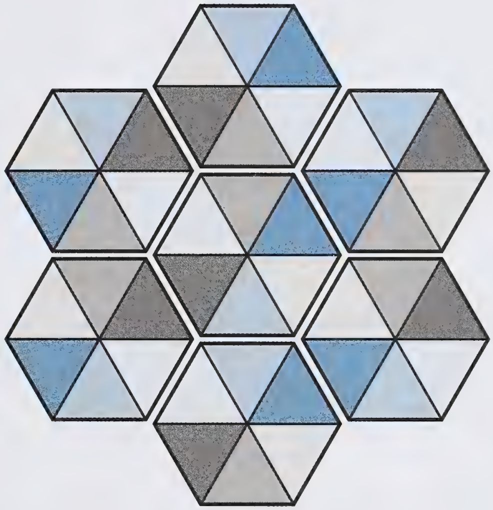
d. You used slides and flips to make your tessellation.

Challenge Activity

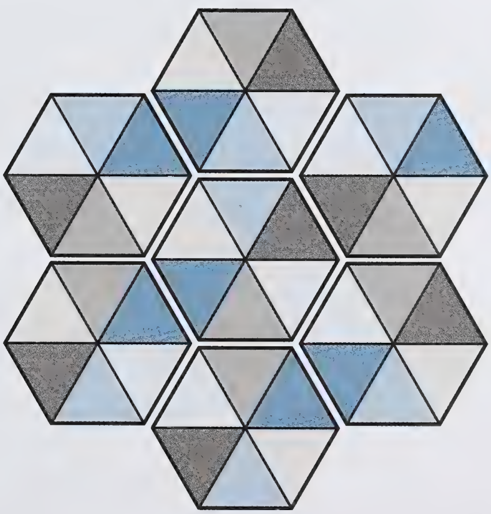
Answers may vary. Sample solutions are given.

The two following solutions are made with slides only.





The following solution is made with slides and turns of the given hexagons.



Lesson 2: Optical Illusions

Activity 1

1. You see a small “moon” (bright circle) on the ceiling.

2. You see a “moon” on the wall that is larger than the one you saw on the ceiling in question 1.

3. The image you saw in question 1 is like looking at the moon when it is overhead in the sky. The image you saw in question 2 is like looking at the moon when it is close to the horizon.

4. **a.** The cross disappears when you hold the picture a certain distance in front of you.
   b. The circle disappears when you hold the picture a certain distance in front of you.

5. **a.** When you get to your blind spot, the cross disappears and your eyes fill in the missing part so that the line looks solid (unbroken).

   b. When you get to your blind spot, the cross disappears and your eyes fill in the missing part. (The circle looks to be filled in completely with blue, even where the cross was.)

6. It appears that you are looking at the object through a hole in your hand.

7. **a.** When you turn it clockwise, the stripes appear to move down.
   b. When you turn it counterclockwise, the stripes appear to move up.

8. **a.** The word CARBON looks flipped over, but the word DIOXIDE looks unchanged.

   b. Each of the letters in the word DIOXIDE has a horizontal line of symmetry, so the word looks the same when flipped. (**Note:** The stem of the glass acts like a solid, round lens that flips images.)

Activity 2

1. **a.** Most people will say that the phrase is “A TRICK FOR THE EYES,” but the actual phrase is “A TRICK FOR THE THE EYES.”

b. It is typical to overlook the extra word “THE” because it is written on a separate line and the triangular frame disguises the repetition.

2. a. Yes, the heavy lines are all parallel.

   b. The small segments crossing the heavy lines draw the eyes in opposite directions. As a result, the heavy parallel lines don’t appear to be parallel to each other.

3. a. The shape is a square, but all four sides appear to bend inward at the middle.

   b. If you used a ruler, you would have discovered that all the sides of the square are perfectly straight. They only look curved because they touch the curves of the circles.

4. a. The picture appears to be nothing more than five irregular polygons, one beside the other in a single row: an octagon, a 14-sided polygon, a 12-sided polygon (dodecagon), a 20-sided polygon (icosagon), and a decagon.

   b. The gaps between the polygons form the word “LIFT.” It may not appear at first because you aren’t accustomed to reading white printing on a darker background, particularly when its border isn’t framed.

5. Answers may vary. Sample answers are given.

   a. Line C appears to be more like a continuation of line A.

   b. If you checked with a ruler, line C is a continuation of line B. The bold vertical lines disguise this fact.

6. Answers may vary. Sample answers are given.

   a. The left side of trapezoid A appears longer than the left side of trapezoid B.

   b. If you checked with a ruler, the left sides of both trapezoids are the same length. This fact is disguised because the right side of trapezoid A is so much longer than the right side of trapezoid B.

7. a. No, the line segment from B to A appears shorter than the line segment from B to C.

   b. If you checked with a ruler, the line segments are the same length. Different things disguise this fact. Point C appears to be farther away because the long parallelogram tilts to the right, in the direction of C. The short segment drawn down from B tilts to the left, in the direction of A.

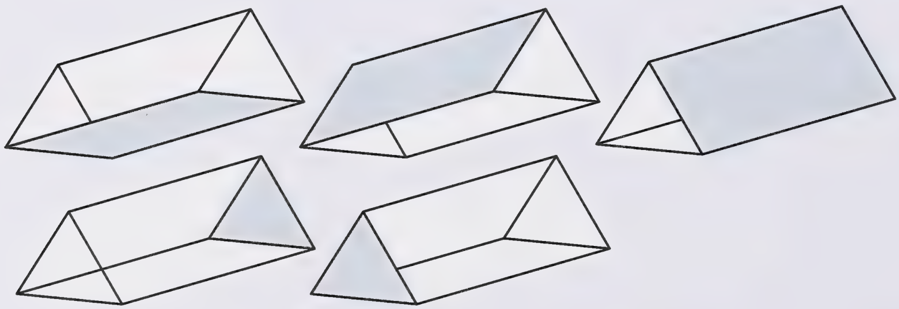
8. In one way, you see the face of a girl. In another way, you see a saxophone player.

9. a. The four shapes that have actually been drawn are three circles (each with a piece missing) and one upward-pointing triangle.

   b. A downward-pointing triangle seems to appear when your eyes try to connect the edges of the missing pieces of the circles. This happens because the missing pieces of the circles are perfectly lined up to represent vertices of a downward-pointing triangle that would be the same size as the upward-pointing triangle.

Activity 3

1. The opening on the cylinder can appear to be on the right or the left, depending on which end you focus on as the opening.

2. 

3. a. The person drawn in the distance appears to be the largest. The person farthest away is drawn to be three panels tall, with his head touching the ceiling. The middle person appears to be about two panels tall, and the person at the front appears to be just over one panel tall.

b. The illusion is caused because the hallway is drawn in perspective using a vanishing point. (The floor tiles and the wall and ceiling panels appear to be the same size, even though their actual dimensions get smaller the further back in the picture they are.) In real life, things of equal size appear smaller the further away they are; however, this is not the case here. By measuring, you will see that the actual drawings of all three people are the same size. Therefore, the people appear taller the further back they are in the picture.

4. You will see two different rectangular prisms, depending on the perspective you take:

- On the right, you will see a prism with eight layers, standing straight up, with a white square on the top. (At the bottom, you will see a white square touching the foot of the prism, and one rectangular wall made from a vertical array of eight rectangles.)

- On the left, you will see a prism with eight layers, tilting upward, and a white square on the bottom. (Behind it, at the top, you will see a white square and one rectangular wall made from a vertical array of eight rectangles.)

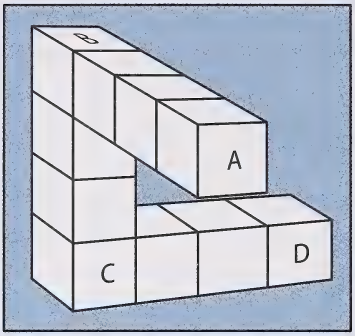
5. The book can appear to be opening either toward you or away from you, depending on which part of the book you focus your eyes on.

6. a. By going from left to right, you would be climbing the dark staircase, so it would take you up.

b. When you rotate the page a quarter-turn counterclockwise, you will see a light staircase that goes down to the right.

7. If you start at the left, you can follow three separate prongs until you get to the base of the fork. At this point, one edge of each of the end prongs is an edge of the centre prong.

8. If you try to build this object using interlocking cubes one row at a time, you will not get a closed triangular shape. (The last cube does not join up with the starting one, as the original picture shows.) Instead, you will get an object like the one shown below.



Start with the right cube (Cube A) and snap on three other cubes straight behind it, ending at Cube B. Then from B, snap on three other cubes straight below it, ending at Cube C. Finally, from Cube C, snap on three other cubes straight to the right of it, ending at Cube D. (It does not end up back at Cube A.)

9. Concentrate just on the U-shaped object. Its right-hand column looks to be in the foreground. Yet, the straight rod is wedged so that the right-hand column of the U-shaped object must be in the background.

Challenge Activity

The hidden objects are highlighted in the following picture.

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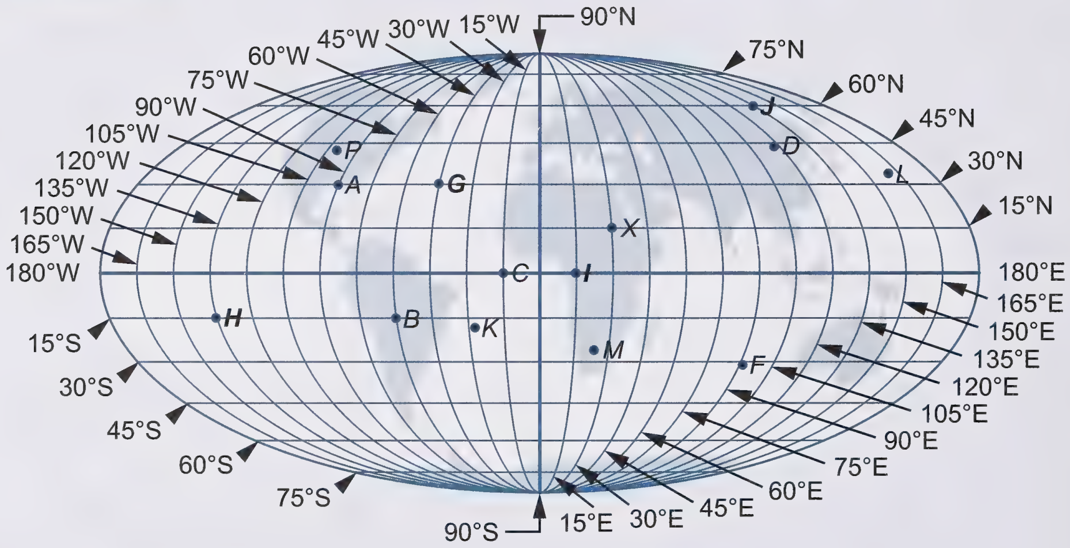
Lesson 3: Plotting Points

Activity 1

1.

Location	Latitude	Longitude
A	30°N	90°W
B	15°S	60°W
C	0°	15°W
D	45°N	120°E
F	30°S	90°E

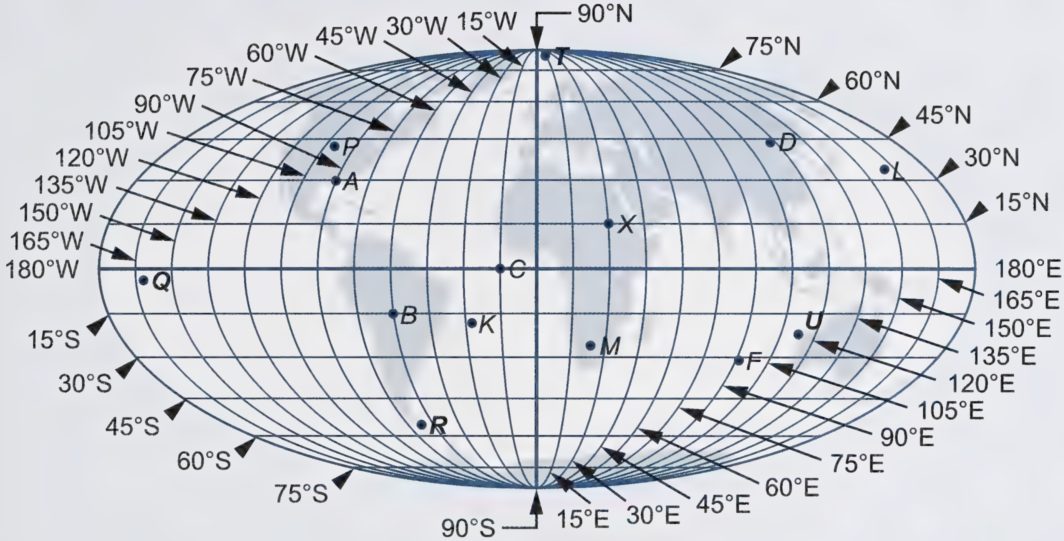
2.



3. Answers will vary. Sample estimates are given.

Location	Estimated Latitude	Estimated Longitude
K	20°S	25°W
L	35°N	160°E
M	25°S	22°E
P	40°N	100°W

4.



5. The coordinates of the corners of a quadrilateral with the least perimeter, with all of South America inside it, are 15°N, 90°W; 15°N, 30°W; 60°S, 90°W; and 60°S, 30°W.

6. If you were at location *C* on the map and you moved 45°N and 90°E, the latitude and longitude of your new location would be 45°N, 75°E.

7. To go from location *F* to location *A* on the map, you would move 60°N and 180°W.

8. **a.** There are four possible ways to go from *C* to *X* along the grid lines, always moving towards *X*: 45°E and 15°N; 15°N and 45°E; 15°E, 15°N, and 30°E; or 30°E, 15°N, and 15°E.

b. Yes, the total distance for each way is always the same because the total distance travelled east is the same and the total distance travelled north is the same.

9. **a.** Fort McMurray: Latitude: about 57°N; Longitude: about 112°W
b. Edmonton: Latitude: about 54°N; Longitude: about 113°W
c. Calgary: Latitude: about 51°N; Longitude: about 114°W

Activity 2

1.

Point	Coordinates
A	(2, 9)
B	(4, 6)
C	(2, 6)

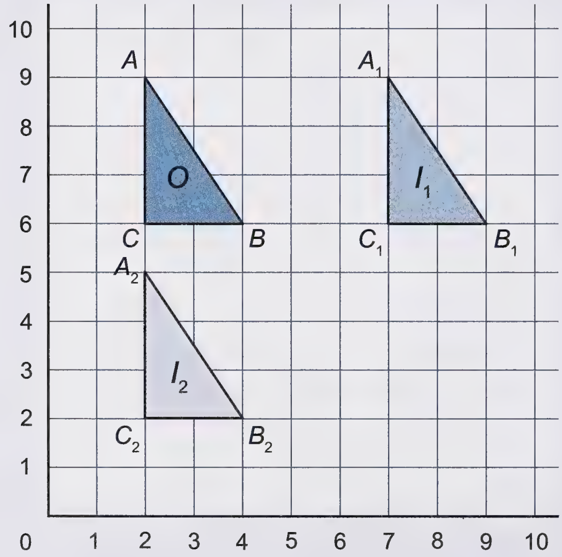
2. The triangle slid five spaces to the right.

3. a. The ordered pairs of the vertices of triangle I_1 are A_1 (7, 9), B_1 (9, 6), and C_1 (7, 6).

   b. You can use the answer from question 2 to answer question 3.a. as follows.

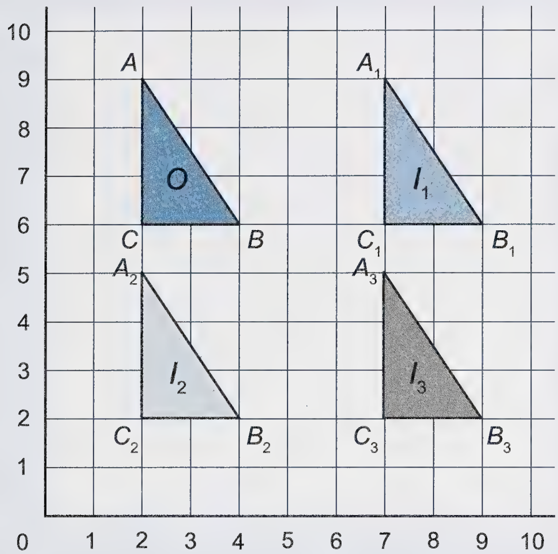
   Each point in the triangle slid five spaces to the right, so the first coordinate in each ordered pair increases by five.

4. a.



   b. The ordered pairs of the vertices of triangle I_2 are A_2 (2, 5), B_2 (4, 2), and C_2 (2, 2).

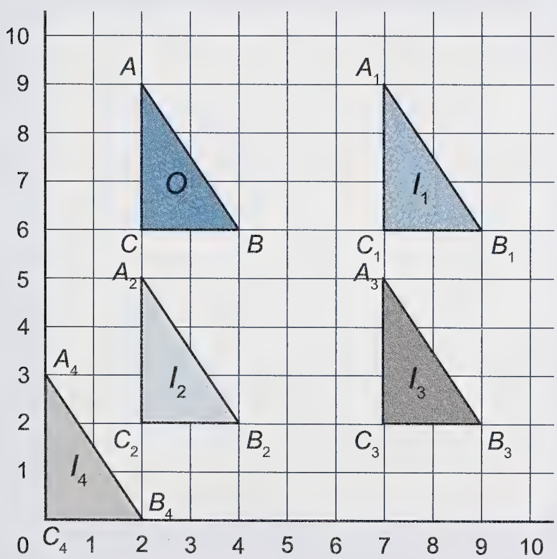
5. a.



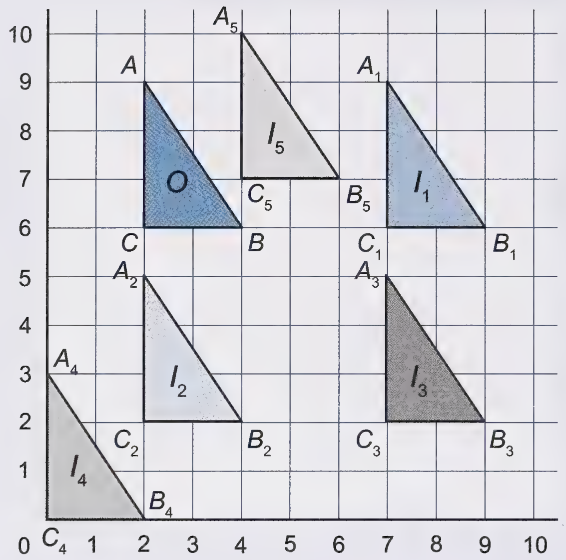
b. The single straight path that slides the original triangle to triangle I_3 is a diagonal that goes to the right five units as it drops down four units.

6. a. Obtain the ordered pairs of the vertices of the slide image by subtracting 2 from each first coordinate and by subtracting 6 from each second coordinate. They become A_4 (0, 3), B_4 (2, 0), and C_4 (0, 0). (**Reminder:** (0, 0) is called the origin.)

b.



7. a.



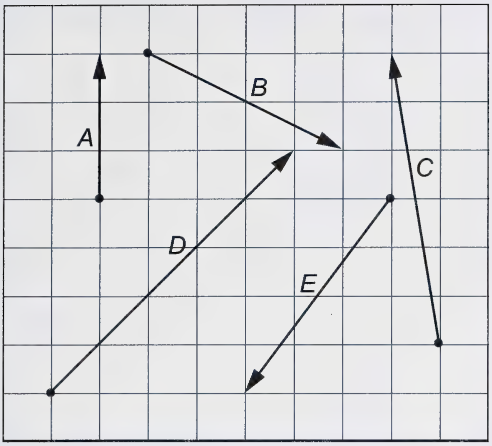
b. The ordered pair of B_5 is (6, 7) and the ordered pair of C_5 is (4, 7).

c. Triangle *ABC* must slide diagonally two units right and one unit up to triangle I_5.

8.

Slide Arrow	Description of Slide
A	Right 3
B	Left 2, Down 1
C	Up 2
D	Left 3
E	Down 1
F	Right 3, Down 2
G	Right 4, Up 3
H	Left 1, Up 1

9.



Activity 3

1.

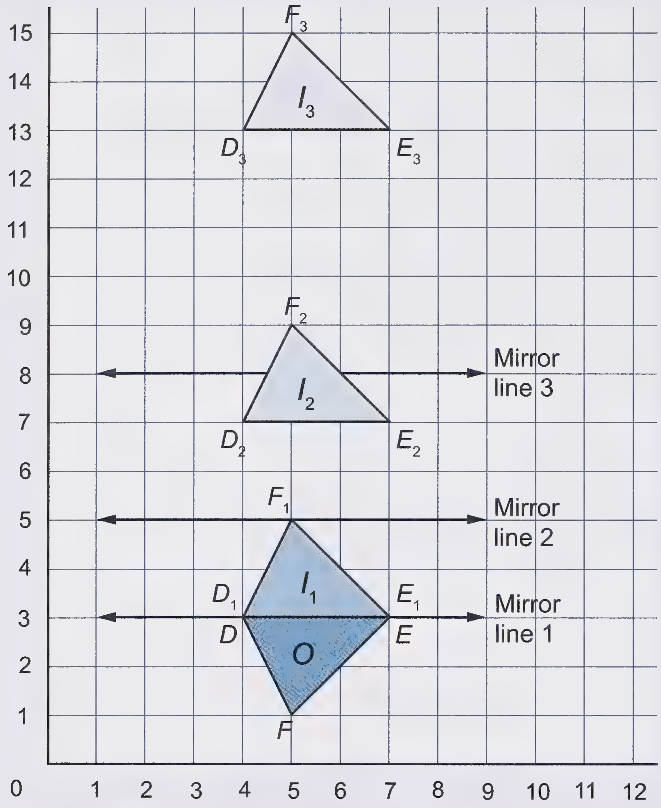
Triangle	ABC	$A_1B_1C_1$	$A_2B_2C_2$	$A_3B_3C_3$
Coordinates of Vertices	A: (11, 5)	A_1: (11, 5)	A_2: (7, 5)	A_3: (3, 5)
	B: (14, 3)	B_1: (8, 3)	B_2: (4, 3)	B_3: (0, 3)
	C: (11, 3)	C_1: (11, 3)	C_2: (7, 3)	C_3: (3, 3)

2. **a.** When a vertical mirror line is used to flip a figure, the second coordinates stay the same. This is because a vertical mirror line does not move the images up or down. It just flips the images horizontally.

   b. A slide moves triangle $A_1B_1C_1$ onto triangle $A_2B_2C_2$ and onto triangle $A_3B_3C_3$.

3. a. The first coordinate in the ordered pair for each vertex will remain the same.

b.



4. a.

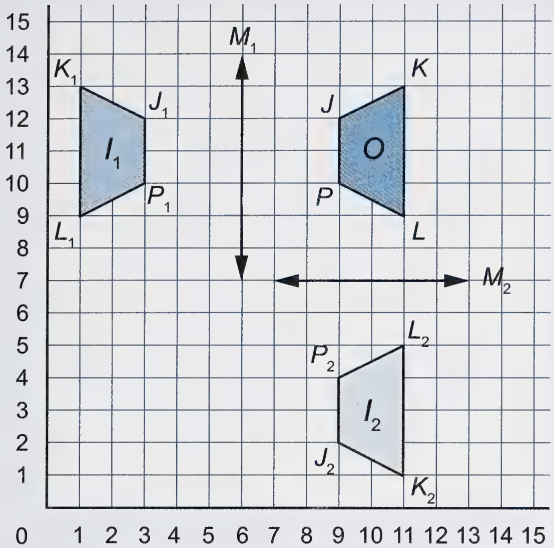
Triangle	DEF	$D_1E_1F_1$	$D_2E_2F_2$	$D_3E_3F_3$
Coordinates of Vertices	D: (4, 3)	D_1: (4, 3)	D_2: (4, 7)	D_3: (4, 13)
	E: (7, 3)	E_1: (7, 3)	E_2: (7, 7)	E_3: (7, 13)
	F: (5, 1)	F_1: (5, 5)	F_2: (5, 9)	F_3: (5, 15)

b. Answers will vary. A sample answer is given.

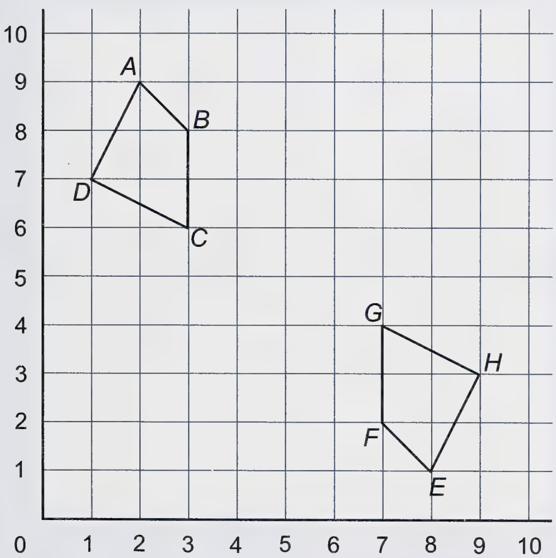
My prediction was correct. This is because the images do not shift horizontally with respect to the original figure.

c. A slide moves triangle $D_1E_1F_1$ onto triangle $D_2E_2F_2$ and onto triangle $D_3E_3F_3$.

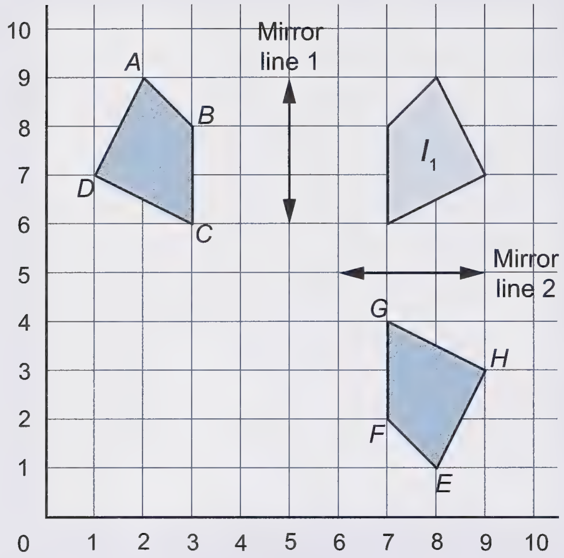
5.



6.

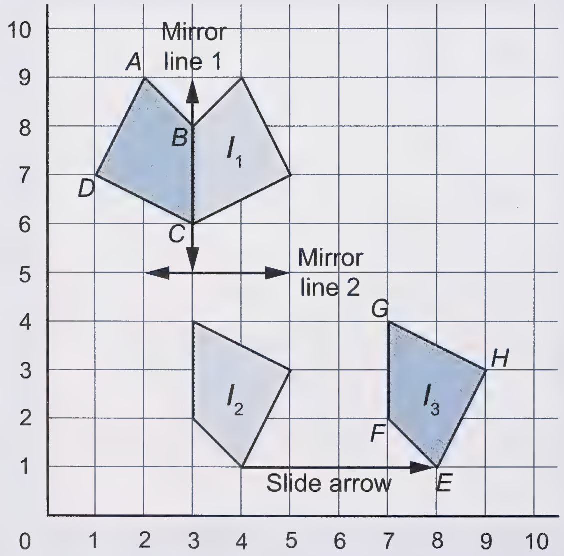


7. a. To move quadrilateral *ABCD* onto quadrilateral *EFGH*, first use a vertical mirror line to flip *ABCD* horizontally to get I_1. Then, use a horizontal mirror line to flip I_1 vertically to get *EFGH*.

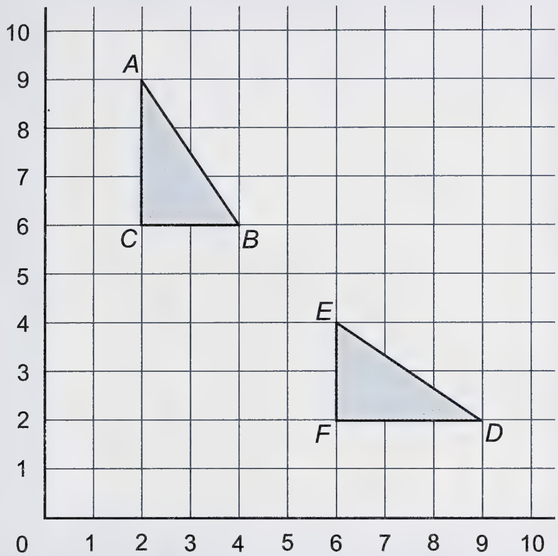


b. Answers will vary. A sample answer is given.

First, use the vertical mirror line to flip *ABCD* horizontally to get I_1. Second, use the horizontal mirror line to flip I_1 vertically onto I_2. Last, slide I_2 right four units to get I_3 and label it *EFGH*.

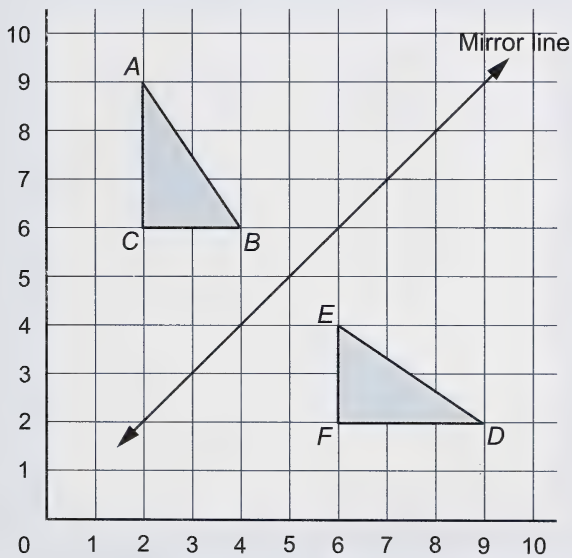


8. a.



b. The coordinates in each of the ordered pairs are switched.

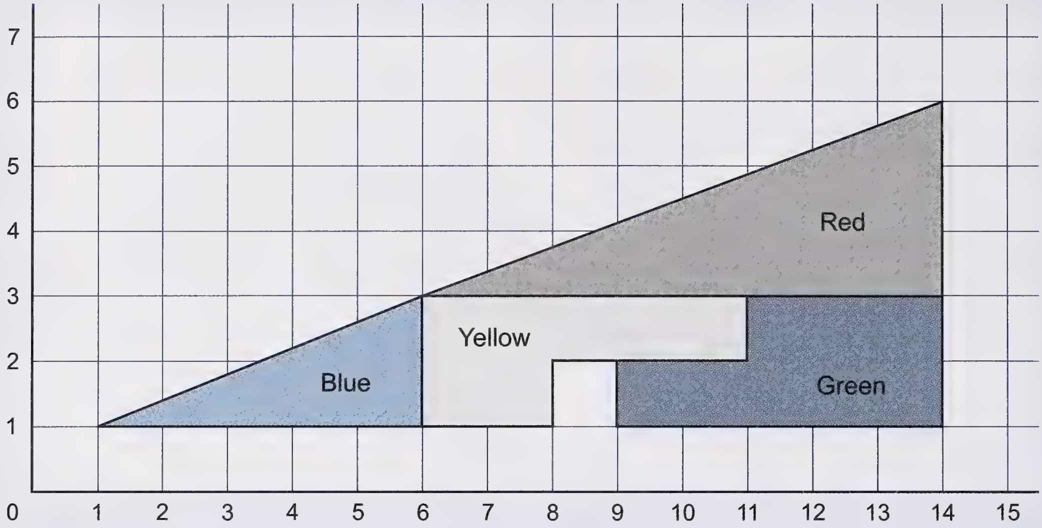
c.



9. No, if only the grid lines can be used as mirror lines, it is not possible to move triangle *ABC* onto triangle *DEF* using a combination of flips and slides. You can first flip *ABC* horizontally using a vertical mirror line and then slide it down, but then you must turn the resulting image to get *DEF*.

Challenge Activity

1.



2. A hole appears in the bottom figure. (One square in the bottom row is empty.) When the four separate pieces are joined, they don't really form two perfect right triangles that are equal in area. Actually, the long diagonal side formed by the slanting blue and yellow edges is not a straight line. It bends slightly down in the first arrangement, but it bends slightly up in the second arrangement (after sliding the pieces). One of the reasons this illusion works is because of the thick black edges. You could see this more easily if the edges were fine lines and if you drew a straight line connecting point (1, 1) with point (14, 6). The difference in the bends causes a difference in the areas of the large triangular-looking figures of 1 square unit, which explains the one small empty square.

Keystrokes

1. a.

Keystrokes	Product Displayed	Upside-down Display
$76 \times 118 =$	8968	8968
$19 \times 472 =$	8968	8968
$59 \times 152 =$	8968	8968
$38 \times 236 =$	8968	8968

b.

Keystrokes	Product Displayed	Upside-down Display
$57 \times 158 =$	9006	9006
$38 \times 237 =$	9006	9006
$79 \times 114 =$	9006	9006
$19 \times 474 =$	9006	9006

c.

Keystrokes	Product Displayed	Upside-down Display
$19 \times 99 =$	1881	1881
$11 \times 171 =$	1881	1881
$33 \times 57 =$	1881	1881
$9 \times 209 =$	1881	1881

d.

Keystrokes	Product Displayed	Upside-down Display
$22 \times 364 =$	8008	8008
$14 \times 572 =$	8008	8008
$56 \times 143 =$	8008	8008
$44 \times 182 =$	8008	8008

2. The products look the same upside down because each four-digit number has half-turn symmetry. Note that each of the digits 1, 0, and 8 forms itself after a turn of 180°, whereas the digits 6 and 9 form each other after a half-turn.

3. The factor pairs for each product in question 1 can be found as follows:

   - Find the prime factorization for each product.
   - Find all the factors by making all possible combinations of the prime factors.
   - Put the factors in pairs by matching lowest with highest, working toward the middle.

4. Answers may vary. The possible factor pairs are given.

   a. Prime factorization: $8968 = 2 \times 2 \times 2 \times 19 \times 59$
   Its factors are 2, 4, 8, 19, 38, 59, 76, 118, 152, 236, 472, 1121, 2242, and 4484.
   Factor pairs not listed are 2×4484, 4×2242, and 8×1121.

   b. Prime factorization: $9006 = 2 \times 3 \times 19 \times 79$
   Its factors are 2, 3, 6, 19, 38, 57, 79, 114, 158, 237, 474, 1501, 3002, and 4503.
   Factor pairs not listed are 2×4503, 3×3002, and 6×1501.

   c. Prime factorization: $1881 = 3 \times 3 \times 11 \times 19$
   Its factors are 3, 9, 11, 19, 33, 57, 99, 171, 209, and 627.
   The factor pair not listed is 3×627.

   d. Prime factorization: $8008 = 2 \times 2 \times 2 \times 7 \times 11 \times 13$
   Its factors are 2, 4, 7, 8, 11, 13, 14, 22, 26, 28, 44, 52, 56, 77, 143, 154, 182, 286, 308, 364, 572, 104, 616, 728, 1001, 1144, 2002, and 4004.
   Factor pairs not listed are 2×4004, 4×2002, 7×1144, 8×1001, 11×728, 13×616, 26×308, 28×286, 52×154, and 77×143.

5. Answers will vary. Sample products and factor pairs are given.

   - $1001 = 7 \times 11 \times 13$, and factor pairs are 7×143, 11×91, and 13×77.
   - $1691 = 19 \times 89$
   - $111 = 3 \times 37$

6. Answers will vary. Sample answers are given.

   - Fourteen keystrokes

Keystrokes	On/C	4	+	4	÷	4	+	4	×	4	+	4	÷	4	=
Display	0	4	4	4	8	4	2	4	6	4	24	4	28	4	7

   - Ten keystrokes

Keystrokes	On/C	4	+	4	+	4	÷	4	+	4	=
Display	0	4	4	4	8	4	12	4	3	4	7

• Ten keystrokes

Keystrokes	On/C	4	+	4	×	4	−	4	÷	4	=
Display	0	4	4	4	8	4	32	4	28	4	7

• Ten keystrokes

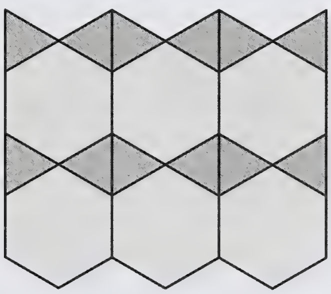
Keystrokes	On/C	4	×	4	−	4	÷	4	+	4	=
Display	0	4	4	4	16	4	12	4	3	4	7

• Seven keystrokes

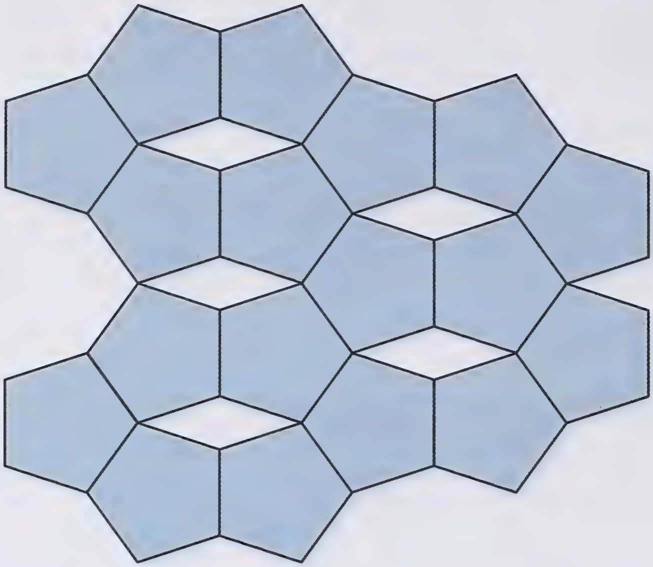
Keystrokes	On/C	4	4	÷	4	−	4	=
Display	0	4	44	44	4	11	4	7

Review

1. Karen's design will tessellate by using only slides. The bottom of the kitten's head (the yellow hexagon) fits in between the ears (the green triangles) without leaving any gaps.

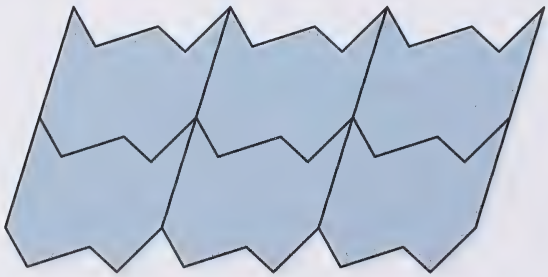


2. You could use a tan parallelogram pattern block to fill in the gaps made when you try to tessellate the pentagon by tracing your cutout repeatedly.

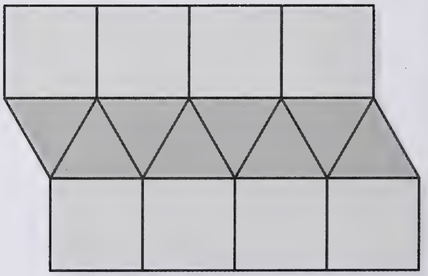


3. a. The new shape will tessellate. The parallelogram used to make it tessellates. The top edges will fit between the bottom edges without leaving any gaps.

   b.



4. Joe's pattern block design will not tessellate by using only slides. The house design must also be flipped.



5. The spokes of the wheel make the small circle appear to be bent (not perfectly round).

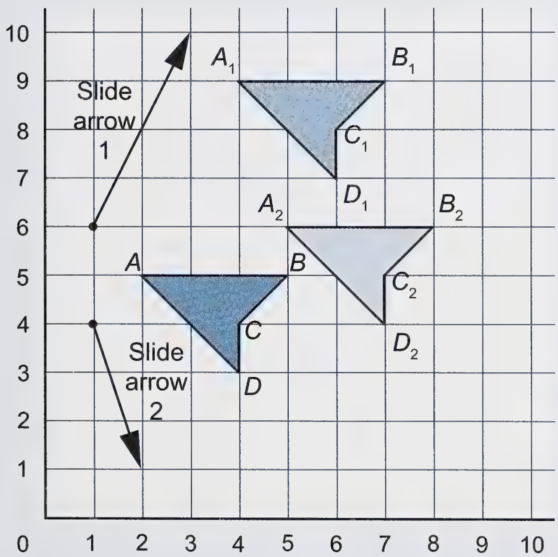
6. You may see a black vase on a white background or you may see two white faces on a black background.

7. One way of looking at the pyramid is by seeing its top point (apex) come up off the page. The other way has its apex going down into the page, like you are looking inside the pyramid. (Its square base looks closer to your eyes.)

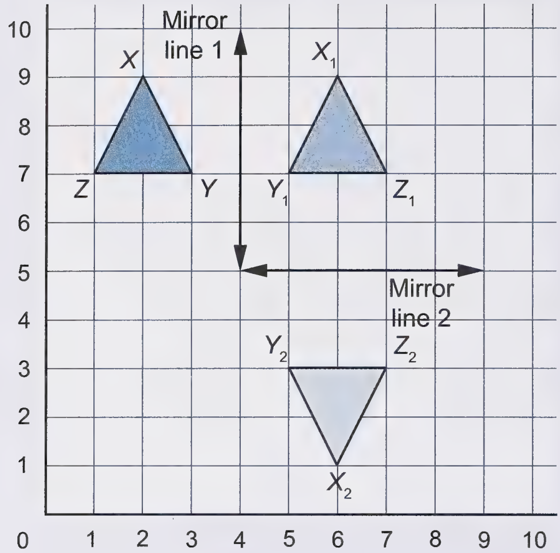
8. It would be impossible to actually make the 3-D object shown because the face with the words *IMPOSSIBLE HOLLOW BAR* appears to be both on the inside of the hollow and on the outside front face of the object.

9. **a.** The square appears to have sides that bulge slightly outward.
b. There is a perfect square between four identical sets of nested circles.
c. The curves of the circles appear to draw the sides of the square outwards.

10.



11.


Mirror line 1
Mirror line 2
X
Y
Z
X1
Y1
Z1
X2
Y2
Z2
0
1
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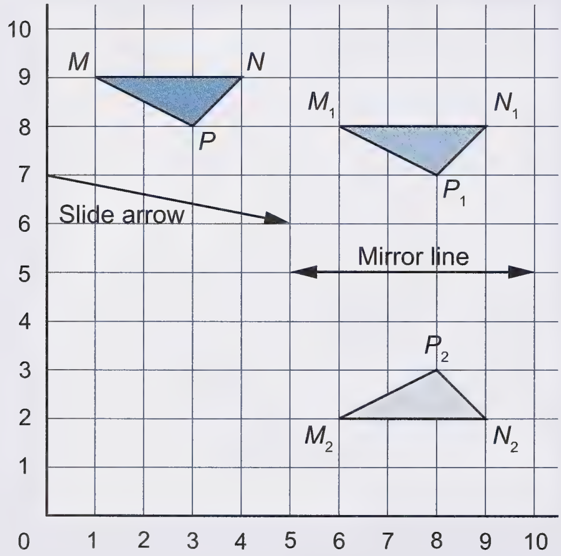
10

12.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N
P
M1
N1
P1
M2
N2
P2
Slide arrow
Mirror line
0
1
2
3
4
5
6
7
8
9
10

Just the Facts

Multiplication and Division Facts

4	7	8	3	56
4	0	3	8	5
0	4	20	1	35
2	2	6	27	0
40	8	54	9	18

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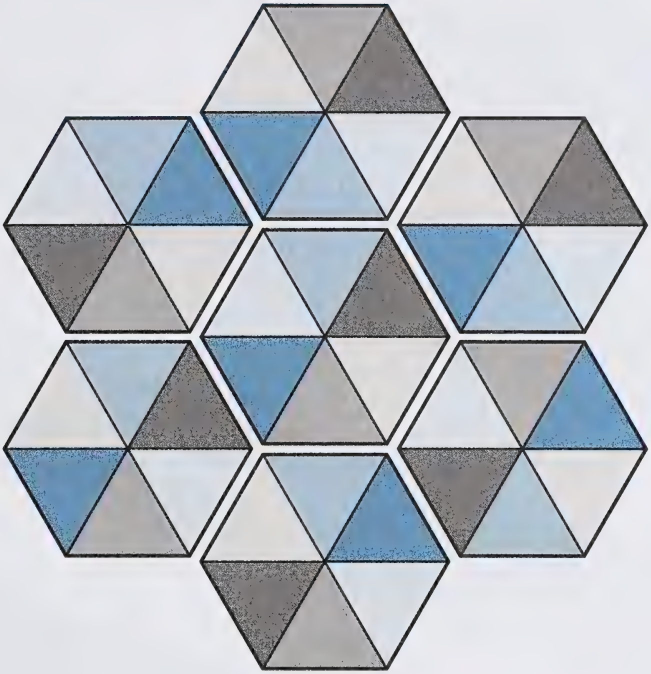
Learning Aids

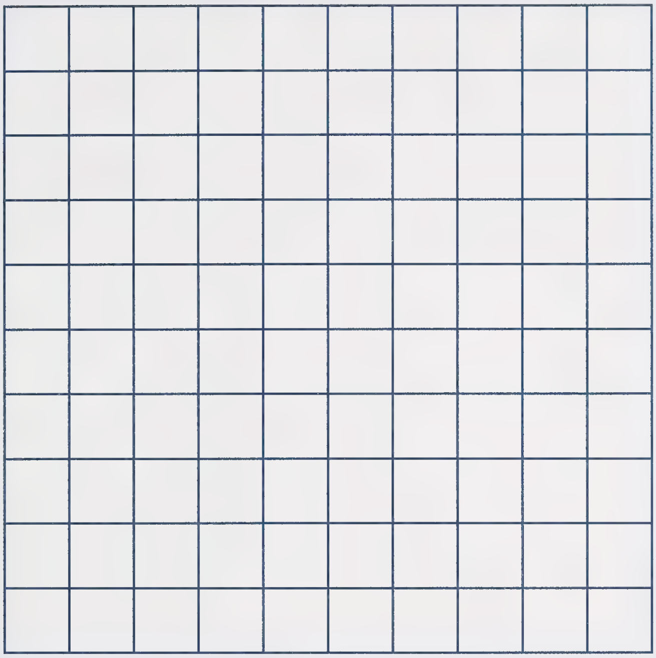
Isometric Dot Paper

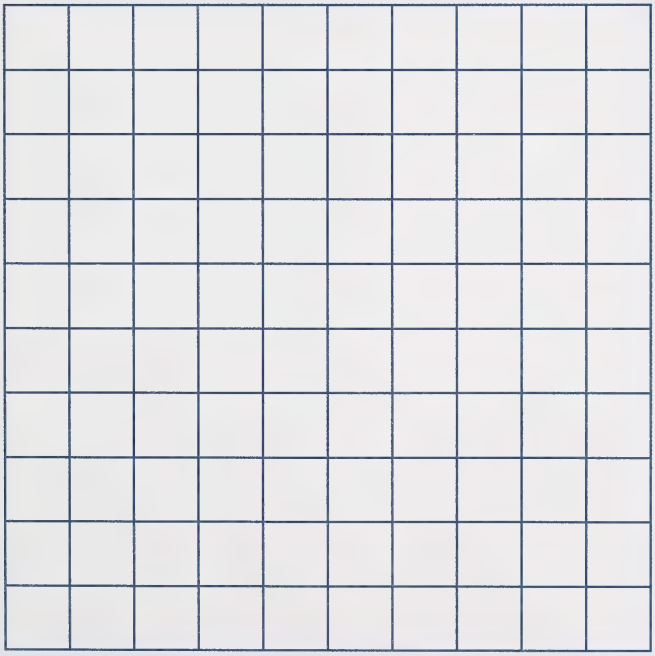
Hexagons for Lesson 1: Challenge Activity



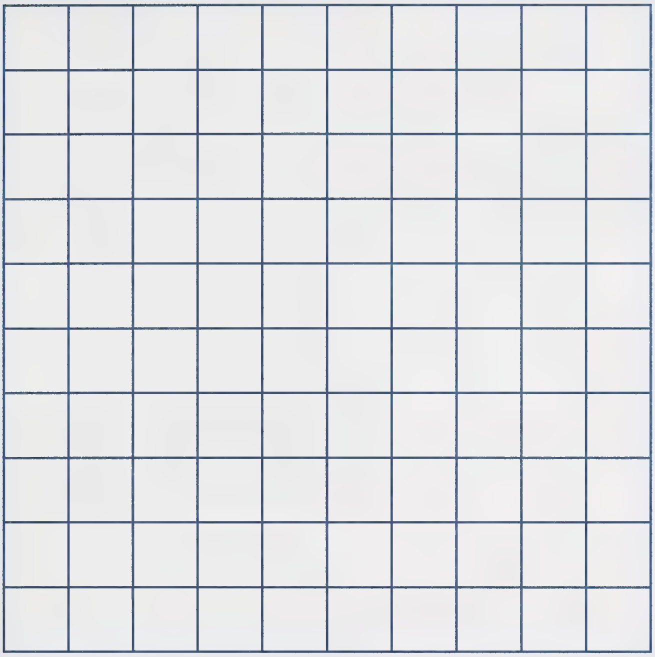
Stripes for Lesson 2: Activity 2

Grids





Grid



Shapes for Lesson 3: Challenge Activity